BC Calculus Final '15-'16
Part I A
No Calculator Allowed

1. The equation of the line normal to the graph of $y=\frac{x}{2x-3}$ at the point $(1, f(1))$ is

$\frac{dy}{dx} = \frac{(2x-3)(1) - x(2)}{(2x-3)^2}$

$m_{TAN} = -3$

$m_{NORM} = 1/3$

(a) $3x+y=4$ (b) $3x+y=2$ (c) $x-3y=-2$

(d) $x-3y=4$ (e) $x+3y=2$

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2. Consider the closed curve in the x-y plane given by $2x^2+5x+y^2+y=8$. Which of the following is correct:

$\frac{d}{dx}(2x^2+5x+y^2+y=8)$

$4x+5+2y\frac{dy}{dx}+\frac{dy}{dx}=0$

(a) $\frac{dy}{dx}=-\frac{4x+5}{8x+2y+1}$ (b) $\frac{dy}{dx}=\frac{4x+5}{2y+1}$

(c) $\frac{dy}{dx}=-\frac{4x+5}{8x+2y}$ (d) $\frac{dy}{dx}=\frac{4x+5}{8x+2y}$

(e) $\frac{dy}{dx}=-\frac{4x+5}{2y+1}$

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3. Which of the following statements are **true**?

I. $\int \left(x^5 \sin x^6\right) dx = -\frac{1}{6}\cos x^6 + c$ II. $\int \tan x\, dx = \sec^2 x + c$

III. $\int \left(\left(x^3 + x\right)\sqrt[4]{x^4 + 2x^2 - 5}\right) dx = \frac{1}{5}\left(x^4 + 2x^2 - 5\right)^{5/4} + c$

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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x	2	5	10	14
$f(x)$	12	28	34	30

4. Let f be a differentiable function on the closed interval [2, 14] and which has values as shown on the table above. Using the sub-intervals defined by the table values and using right hand Riemann sums, $\int_2^{14} f(x)dx =$

a) 296

$3(28) + 5(34) + 4(30)$

b) 312

c) 343

d) 374

e) 390

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5. The function f is defined for all Real numbers such that

$$f(x)=\begin{cases} x^2+kx & \text{for } x<3 \\ 5\sin\frac{\pi}{2}x & \text{for } x\geq 3 \end{cases}$$

$x=3 \Rightarrow 9+3k=-5$

$3k=-14$

For which value of k will the function be continuous throughout its domain? $k=\frac{-14}{3}$

(a) $-\frac{14}{3}$ (b) $-\frac{1}{6}$ (c) $\frac{4}{3}$ (d) 1 (e) None of these

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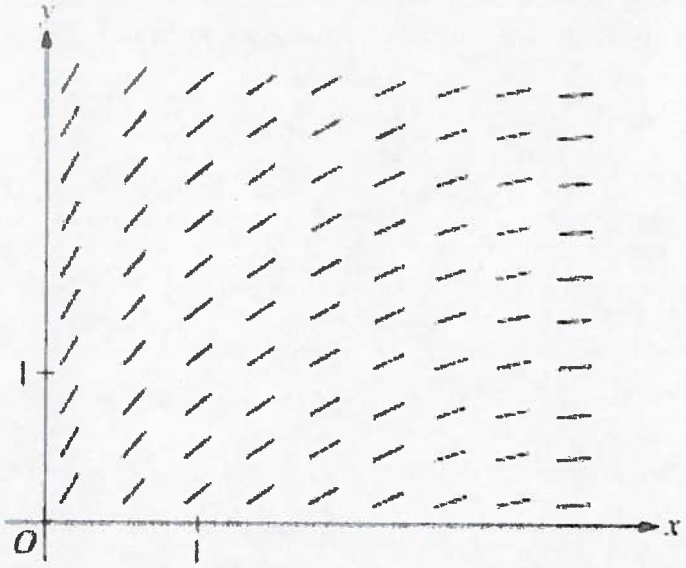
6. $2\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2e^{\sqrt{x}}+c$ $\quad u=\sqrt{x}$ $\quad du=\frac{1}{2}x^{-1/2}dx$

a. $2e^{\sqrt{x}}+c$ b. $\frac{1}{2}e^{\sqrt{x}}+c$ c. $e^{\sqrt{x}}+c$

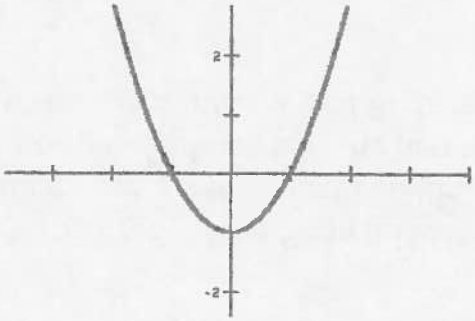
d. $2\sqrt{x}\,e^{\sqrt{x}}+c$ e. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}+c$

7.



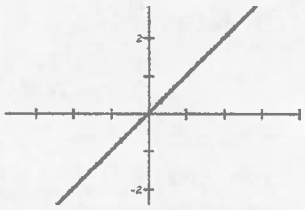
The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$



8. Suppose the derivative of f has the graph shown above. Which of the following could be the graph of f?

A)



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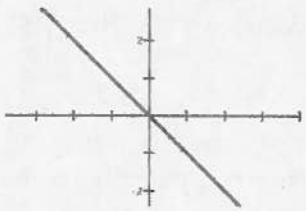
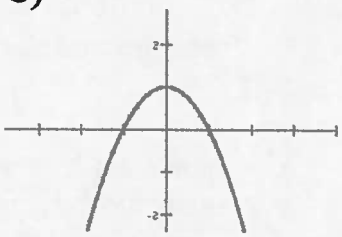
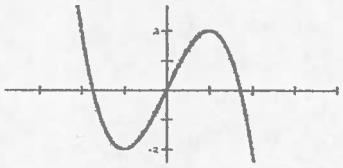


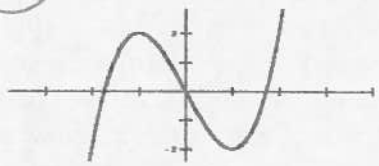
C)



D)



E)



9. On which of the following intervals is the graph of the curve $y = x^5 - 5x^4 + 10x + 15$ concave up?

$y' = 5x^4 - 20x^3 + 10$

$y'' = 20x^3 - 60x^2 = 20x^2(x-3)$

I. $x < 0$ II. $0 < x < 3$ III. $x > 3$

y'': 0 | $-$ | 0 | $+$ (at x, 0, 3)

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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10. If $y = x^2 \cos 2x$, then $\frac{dy}{dx} =$ $x^2(-2\sin 2x) + 2x\cos 2x$

$2x(\cos 2x - x\sin 2x)$

(A) $-2x\sin 2x$

(B) $-4x\sin 2x$

(C) $2x(\cos 2x - \sin 2x)$

(D) $2x(\cos 2x - x\sin 2x)$

(E) $2x(\cos 2x + \sin 2x)$

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11. $\lim_{x\to 1}\frac{e^x-1}{\ln x}=$ $\frac{e-1}{0}=$ DNE

a) 0 b) e c) $\frac{1}{e}$ d) $-e$ e) Undefined

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12. Find the average rate of change of $y=x^2+5x+14$ on $x\in[-1,2]$

a) 3 b) 6 c) 9 d) $\frac{65}{6}$ e) 18 $\frac{1}{2-(-1)}\int_{-1}^{2}(x^2+5x+14)\,dx$

$\frac{1}{3}\left[\frac{x^3}{2}+\frac{5x^2}{2}+14x\right]_{-1}^{2}$

13. $\lim_{h\to 0}\dfrac{\cos\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}\right)}{h}=$ $\frac{d}{dx}\cos x\Big|_{x=\pi/6} = -\sin\frac{\pi}{6}$

a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) $\frac{\sqrt{3}}{2}$

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14. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x)=g(g(x))$, $h'(8)=$ $g'(g(8))\cdot g'(8) = g'(2)\cdot g'(8) = 1\,(4)$

a) 1 b) 2 c) 3 d) 4 e) 8

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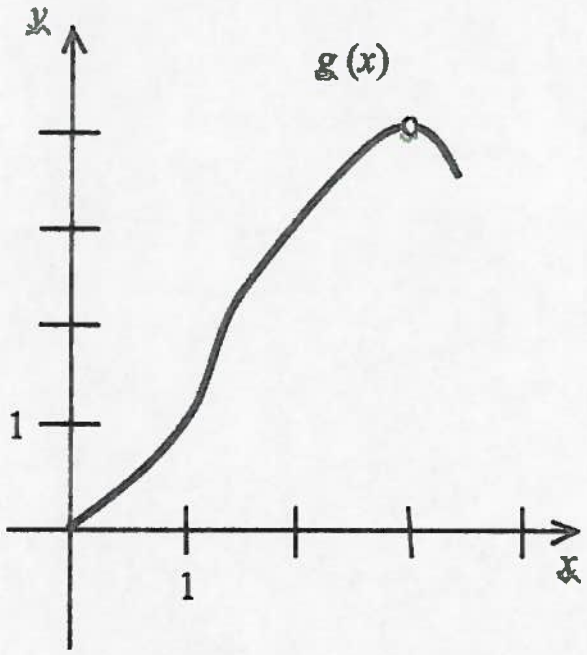
15. The graph of a function g is shown below. Which of the following statements are TRUE?



I. $\lim_{x\to 3} g(x) = 4$ II. $\lim_{h\to 0} \frac{g(2+h)-g(2)}{2} > 0$ III. $g(3)$ does not exist

$g'(2) > 0$

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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16. State the step that has the first mistake in this process:

$$\int \frac{x}{3x^4+4}\,dx =$$

Step 1: $\frac{1}{12}\int \frac{12x}{3x^4+4}\,dx =$

Step 2: $\frac{1}{12}\int \left(\frac{12x}{3x^4}+\frac{12x}{4}\right)dx =$

Step 3: $\frac{1}{12}\int \left(\frac{4}{x^3}+3x\right)dx =$

Step 4: $\frac{1}{12}\int \left(\frac{4}{x^3}\right)dx+\frac{1}{12}\int (3x)\,dx =$

Step 5: $-\frac{1}{3x^2}+\frac{1}{4}x^2+c$

a) Step 2
b) Step 3
c) Step 4
d) Step 5
e) There is no mistake.

17. Which of the following functions fail to meet the conditions of the Mean Value Theorem?

I. $3x^{2/3} - 1$ on $[-1, 2]$ NOT DIFF @ 0

II. $|3x - 2|$ on $[1, 2]$

III. $4x^3 - 2x + 3$ on $[-1, 2]$

NOT DIFF @ $x = 2/3$ BUT NOT IN DOMAIN

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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18. If the radius of a sphere is increasing at 2 in/second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

a. 40π b. 80π c. 800

d. 800π e. 3200π

$V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi (10)^2 (2)$

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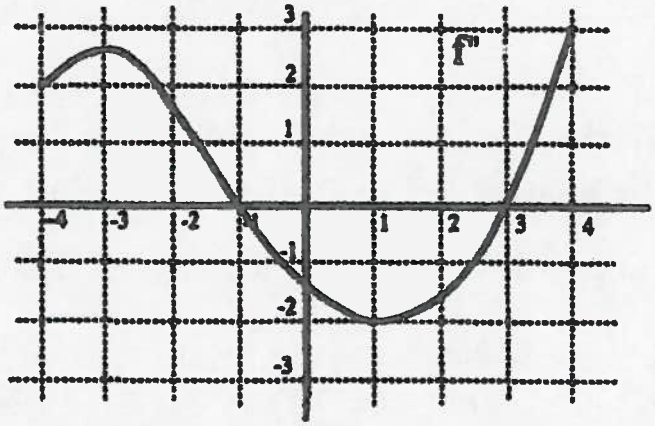
19. The graph of the **second derivative** of a function f is shown at right. Which of the following is false?

the graph of f''



I. The graph of f has an inflection point at $x = -1$. TRUE

II. The graph of f is concave down on the interval $(-1,3)$. TRUE

III. The graph of the derivative function f' is increasing at $x = 1$. FALSE

$f'' > 0$ no!

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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20. Consider the function $f(x)=\frac{x^4}{2}-\frac{x^5}{10}$. $f'(x)$, the *derivative* of f, attains its maximum value at $x=$

$f' = 2x^3 - \frac{1}{2}x^4$; $f'' = 6x^2 - 2x^3 = 0$

$2x^2(3-x) = 0$

a) 0 b) 3 c) 4 d) 5 e) Never

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21. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, then $\int_{-5}^{5} f(x)\,dx =$

$\int_{-5}^{2} + \int_{2}^{5}$

a) −21 b) −13 c) 0 d) 13 e) 21

$= -17 - (-4)$

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22. The acceleration of a particle is given by $a(t)=4e^{2t}$. When $t=0$, the position of the particle is $x=2$ and $v=-2$. Determine the position of the particle at $t=\frac{1}{2}$.

a) $e-3$
b) $e-2$
c) $e-1$
d) e
e) $e+1$

$v=\int 4e^{2t}\,dt = 2e^{2t}+C$

$2+C=-2$

$C=-4$

$x=\int 2e^{2t}-4 = e^{2t}-4t+C_2$

$1-0+C_2=2 \quad C_2=1$

$x=e^{2t}-4t+1$

$e-2+1$

23. Let g be the function defined by $\int_{-1}^{x} \frac{t^2-2t-15}{\sqrt{t^2+4}}\,dt$. On which intervals is g decreasing?

a) $x \le -3$ or $5 \le x$
b) $-3 \le x \le 5$
c) $x \le 1$
d) $1 \le x$
e) none of these

$\frac{dy}{dx} = \frac{x^2-2x-15}{\sqrt{x^2+4}} = \frac{(x-5)(x+3)}{\sqrt{x^2+4}}$

y' + 0 − 0 +
c
x −3 5

---

24. Which of the following series diverge?

I. $\sum_{n=3}^{\infty} \frac{3n^5}{7n^4-1}$ DIV II. $\sum_{n=0}^{\infty} \frac{7^{n+1}}{(2\pi)^n}$ $r = \frac{7}{2\pi} > 1$ DIV III. $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{5^n}$ AST

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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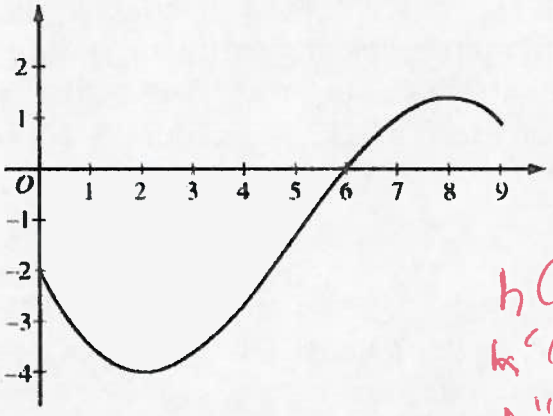
25. $\lim_{x\to\pi} \frac{\int_{\pi}^{x} (\cos^2 t)\,dt}{\sin 2x} =$ L'H $\lim_{x\to\pi}$ $\frac{\cos^2 x}{\cos 2x\,(2)} = \frac{1}{2}$

a) -1 b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 1

BC Calculus Final '14-'15
Part I B
Calculator Allowed



Graph of f

$h(6) < 0$
$h'(6) = 0$
$h''(6) > 0$

1. The graph of differentiable equation f is shown above. If $h(x)=\int_0^x f(t)dt$, which of the following is true?

a) $h(6)<h'(6)<h''(6)$
b) $h(6)<h''(6)<h'(6)$
c) $h'(6)<h(6)<h''(6)$
d) $h''(6)<h(6)<h'(6)$
e) $h''(6)<h'(6)<h(6)$

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2. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)dx$? = Degrees

a) The change in temperature during the first t hours.
b) The change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24th day.

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3. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1+2^t}$, where V is the volume of the ice in cubic feet and t is the time in minutes. The amount of ice which has melted in the first five minutes is

a) $14.49\ ft^3$ b) $14.51\ ft^3$ c) $14.53\ ft^3$

d) $14.55\ ft^3$ e) $14.57\ ft^3$

---

4. Which of the following improper integrals diverge?

I. $\int_0^{\infty} \frac{2x}{1+x^2}\,dx$ II. $\int_0^{9} \frac{1}{\sqrt{x}}\,dx$ III. $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

a) I only b) II only c) III only d) I and II only e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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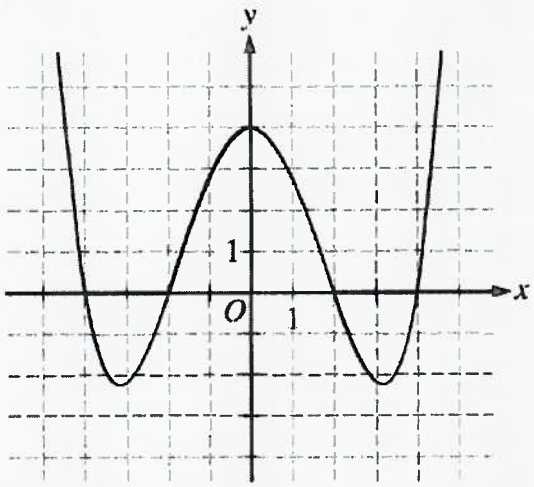
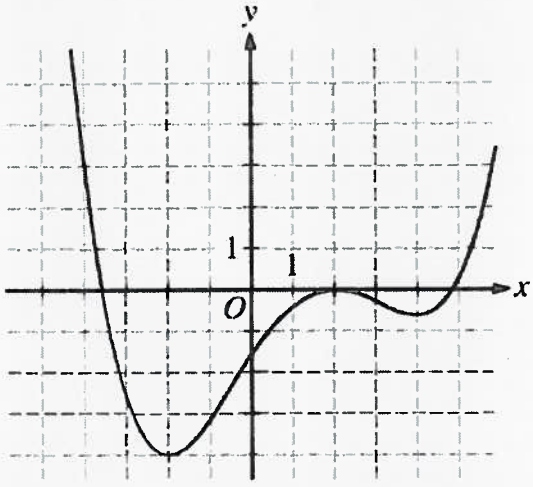
5. The average value of $y = e^x \cos x$ on $x \in \left[0, \frac{\pi}{2}\right]$ is

a. 0 b. 1.213 c. 1.905 d. 2.425 e. 3.810



Graph of f



Graph of g

6. The graphs of the differentiable functions $f(x)$ and $g(x)$ are shown above. If $P(x)=f(x)g(x)$, which of the following will be **true** about P'?

a) $P'(2)<0$

b) $P'(2)=0$

c) $P'(2)>0$

d) $P'(0)<0$

e) $P'(0)=0$

$P' = f \cdot g' + g \cdot f'$

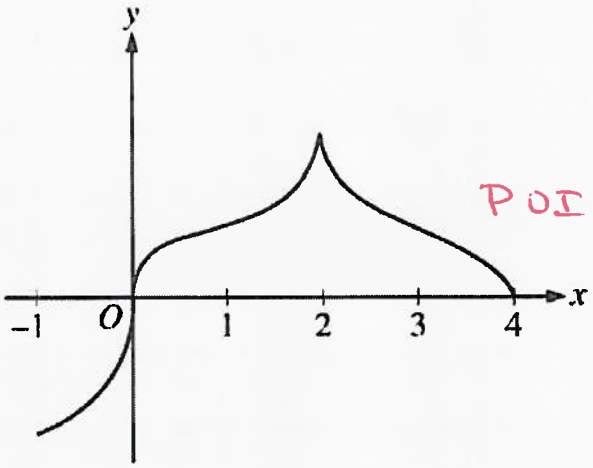
$= 0 \cdot 0 + 0(-)$

$= 0$

7. A particle is moving in a straight line with velocity $v = 7 - (1.01)^{-t^2}$. The acceleration at $t = 3$ is

a. –0.914 b. 0.055 c. 5.486 d. 6.086 e. 18.087

MATH 8 $v'(3) =$



POI @ x=2

Graph of f'

8. The graph of f' is shown above. The line tangent to f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is TRUE?

a) f' does not exist at $x = 2$.

b) f is decreasing on the interval (2, 4)

c) The graph of f has a point of inflection at $x = 0$

d) The graph of f has a relative maximum at $x = 0$

e) The graph of f has a point of inflection at $x = 2$

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9. Let $y=f(x)$ define a twice-differentiable function and let $y=t(x)$ be the line tangent to the graph of f at $x=2$. If $t(x)\le f(x)$ for all real numbers, which of the following must be true?

a) $f(2)\le 0$

b) $f'(2)\le 0$

c) $f'(2)\ge 0$

d) $f''(2)\le 0$

e) $f''(2)\ge 0$

$t<f$ MEANS THE TAN LINE IS UNDER THE CURVE $\therefore$ f IS CONCAVE UP.

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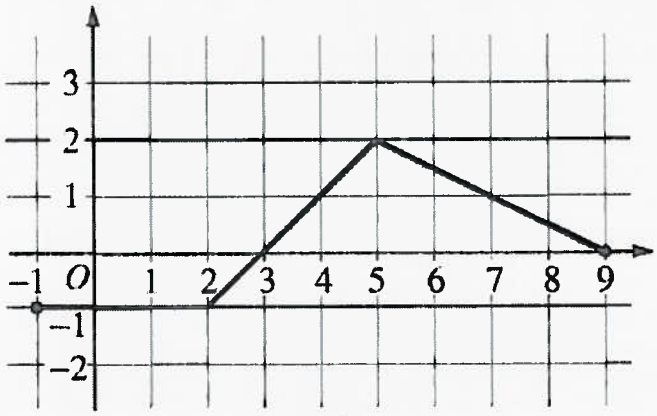
10. If the graph of a function f has a vertical asymptote at $x=3$, which of the following must be false?

a) $\lim_{x\to 3} f(x)=0$ b) $\lim_{x\to 3} f(x)=-\infty$ c) $\lim_{x\to 3} f(x)=\infty$

d) $\lim_{x\to \infty} f(x)=3$ e) $\lim_{x\to \infty} f(x)=\infty$



Graph of f

11. The graph of a piecewise linear function f is shown above. Let h be the function given by $h(x)=\int_{-1}^{x} f(t)\,dt$. On which of the following intervals is h decreasing?

h IS DECREASING WHERE f ≤ 0

a) $[-1,3]$ b) $[-1,5]$ c) $[2,5]$

d) $[5,9]$ e) $[3,9]$

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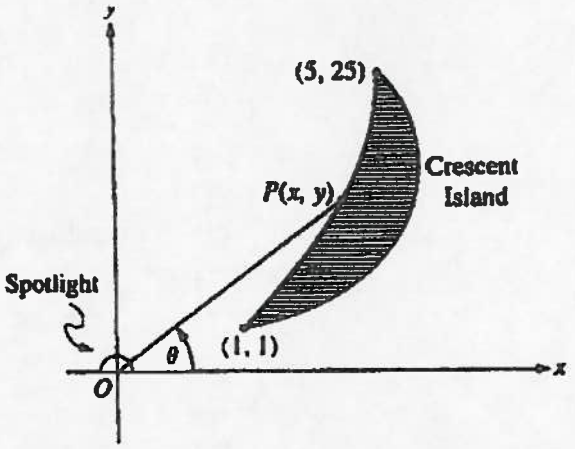
12. Let f be a function such that $f'(x)=\ln x\cos x+\frac{\sin x}{x}$. In the interval $0<x<3$, the graph of f has a point of inflection nearest $x=$

a) 0.352 b) 1.101 c) 2.128 d) 2.259 e) 2.901



$\tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{25}{5}\right) = 1.373$

13. The figure above shows a spotlight shining on a point $P(x,y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shine is in the shape of the parabola $y=x^2$ from the point (1, 1) to (5, 25). Let be the angle between the beam of light and the positive x-axis. For what values between 0 and $\frac{\pi}{2}$ does the spotlight shine on the shoreline? BC #77

a) $.5 \le x \le .785$

b) $.785 \le x \le 1.190$

c) $1.190 \le x \le 1.373$

d) $.785 \le x \le 1.373$

e) $1.373 \le x \le 1.570$

14. The function f is continuous and twice differentiable on the interval (2,4). If $f'(3)=2$ and $f''(x)<0$ on the interval, which of the following could be a table of values for $f(x)$? INCREASING & CONCAVE DOWN

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

TAN LINE $\rightarrow y = 2(x-3)+5$

$x=4 \rightarrow y<7$

$x=2 \rightarrow y<3$

15. The number of parts per million (ppm), $C(t)$, of chlorine in a pool changes at the rate of $C'(t)=1-3e^{-0.2\sqrt{t}}$ ounces per day, where t is measured in days. There are 10 ppm of chlorine in the pool at time $t = 0$. How many ounces of chlorine are in the pool when $t = 9$?

a) -.646 b) 9.354 c) -.652

d) 9.285 e) 0.715 $10+\int_0^9 C'(t)=$

---

16. Which of the following converge?

I. $\left\{\frac{2+n^2}{4+n^3}\right\}$ II. $\left\{\frac{2+n^3}{1+3n^3}\right\}=\frac{1}{3}$ III. $\left\{\frac{2+n^4}{1+3n^3}\right\}=\infty$

a) I only b) II only c) III only d) I and II only e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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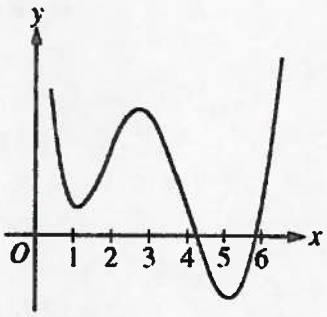
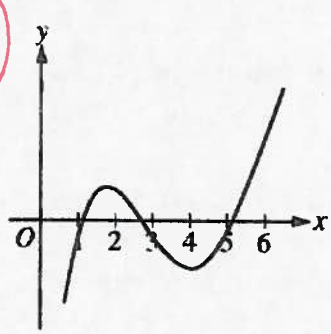
x	1	2	3	4	5	6
$f(x)$	1	3	4	1	−2	1

17. The function f is twice differentiable with selected values given in the table above. Which of the following cold be the graph of $f''(x)$, the second derivative of f?

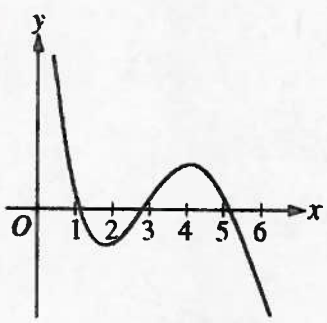
(A)



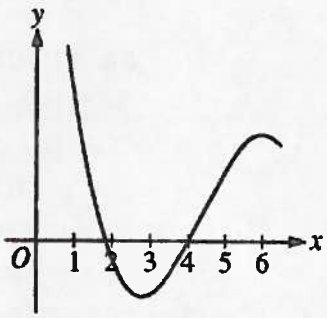
(B)



(C)



(D)



(E)