AP Calculus BC ’15-16

Spring Final Part IIA

Calculator Allowed

Name:

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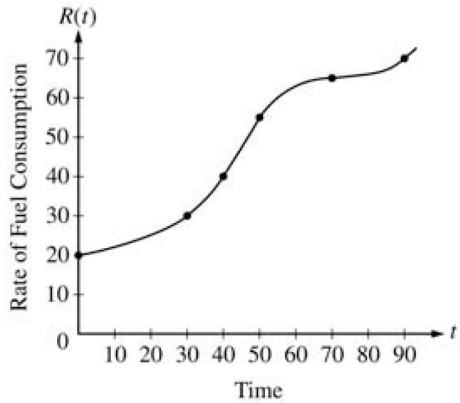


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

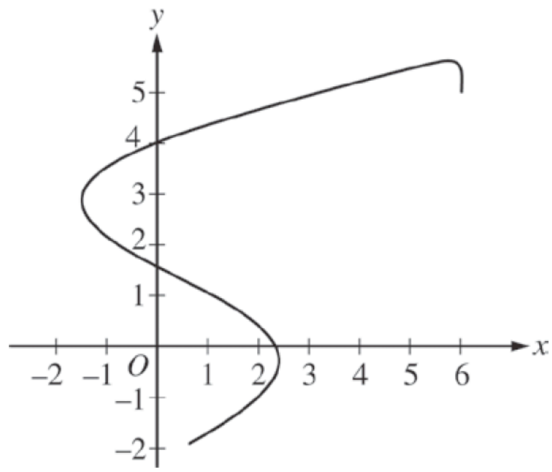
1. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice differentiable and strictly increasing function $R(t)$. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use the data on the table to approximate $R'(45)$. Show the computations that lead to your answer. Indicate the units.

(b) The rate of fuel consumption is increasing fastest at $x = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)dt$ using a left Riemann sum with the five subintervals indicate by the data in the table. Is this numerical approximation less than the actual value of $\int_0^{90} R(t)dt$? Explain your reasoning.

(d) For $0 \le b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)dt$ in terms of fuel consumption by the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)dt$ in terms of fuel consumption by the plane. Indicate units of Measurement for both answers.



2. An Planetary rover travels on a flat surface. The path for the rover for the time interval $0 \le t \le 2$ hours is shown above. The rover starts with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position change at rates given by

$$x'(t) = -12\sin(2t^2) \text{ and } y'(t) = 10\cos(1+\sqrt{t}),$$

where $(x(t), y(t))$ are measured in meters and t is measured in hours.

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a. Find the acceleration vector of the rover at $t = 1$. Find the speed at $t = 1$.

b. Find the total distance traveled over the time interval $0 \le t \le 1$.

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c. Find the y-coordinate of the position of the rover at time $t = 1$.

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d. The rover receives a signal at each point where the tangent line to its path has slope $\frac{1}{2}$. At what times t, of $0 \le t \le 2$, does the rover receive the signal?

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NO Calculator Allowed

Name:

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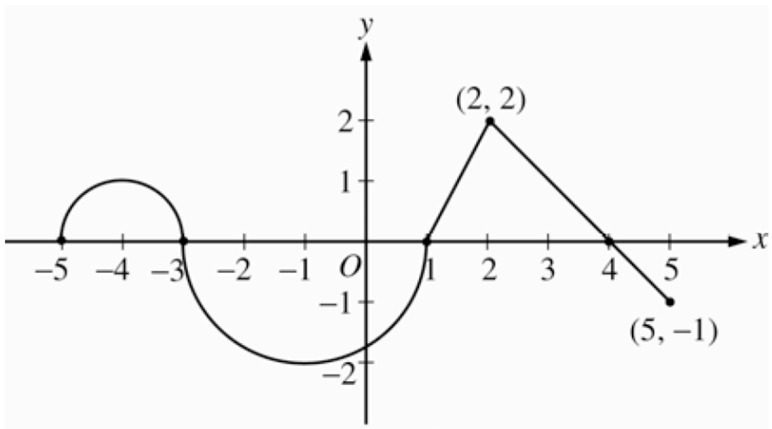
3. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown below.



Graph of f'

(a) For $-5 < x < 5$, find all the values x at which the graph of f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all the values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also increasing. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

4. In a national park, the population of mountain lions grows over time. At $t = 0$, where t is measured in years, the population is found to be 20 mountain lions.

(a) One zoologist suggests a population model P that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$. Use separation of variables to solve the differential equation for P with the initial condition above.

(b) A second zoologist suggests a population model Q that satisfies the $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when Q grows most rapidly.

(c) For the population Q introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.

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5. Consider the function $f(x) = x^2 \sin(kx)$, where k is a nonzero constant.

(a) Let $k = 2$. Write an equation for the line tangent to the graph of f at the point whose x-coordinate is $\frac{\pi}{2}$.

(b) Let $k = \frac{1}{2}$, so that $f(x) = x^2 \sin\left(\frac{1}{2}x\right)$. Find $f'(x)$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

(c) Let $k=3$, so that $f(x)=x^2\sin(3x)$. Use integration by parts to solve $\int x^2\sin(3x)dx$.

6. The function $f(x)$ satisfies the equation

$$f'(x) = f(x) + x + 1.$$

And $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all *x*.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second degree Taylor polynomial for *f* about $x = 0$.

(c) Find the fourth degree Taylor polynomial for *f* about $x = 0$.

(d) Find $f^n(0)$, the nth derivative of *f* about $x = 0$, for $n \geq 0$. Use the Taylor series for *f* about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) - 4e^x$.

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