AP Calculus BC '16-17
Anti Derivative Test

Name SOLUTION KEY

Score___________

1. Which of the following statements are true?

I. $\int\left(\sin^3 x\ \cos^2 x\right) dx = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + c$

~~II.~~ $\int \sec 2x\, dx = 2\sec 2x \tan 2x + c$ THIS IS THE DERIVATIVE, NOT $\int$

~~III.~~ $\int\left(\frac{3x^2+6x-4}{\left(x^3+3x^2-4x+2\right)^2}\right) dx = \ln\left|x^3+3x^2-4x+2\right|^2 + c$ NO $\int \frac{1}{u}\,du$

a) I only b) II only c) III only

d) I and II only e) II and III only

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2. $\int \frac{x^2-1}{x} dx = \int\left(x - \frac{1}{x}\right) dx = \frac{x^2}{2} - \ln|x| + c$

a) $\frac{1}{2}\ln\left|x^2-1\right| + C$ b) $\frac{1}{2}\left(x^2-1\right)^2 + C$ c) $\frac{x^2}{2} - \ln|x| + C$

d) $x - \frac{1}{x} + C$ e) $1 + x^{-2} + C$

$y = -\int u^3 du = -\frac{1}{4}\cos x + c \quad (\pi, 1) \Rightarrow 1 = \frac{(1)}{4} \pm c \Rightarrow c = 5/4$

3. If $\frac{dy}{dx} = \sin x \cos^3 x$ and if $y = 1$ when $x = \pi$, what is the value of y when $x =$ 0?

$y(0) = -\frac{1}{4}\cos(0) + 5/4$
$= 1$

a) -2 b) -1 c) 0 d) 1 e) 2

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4. $2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = 2\int e^u du = 2e^{\sqrt{x}} + c$

a) $2e^{\sqrt{x}} + c$ b) $\frac{1}{2}e^{\sqrt{x}} + c$ c) $e^{\sqrt{x}} + c$

d) $2\sqrt{x}\; e^{\sqrt{x}} + c$ e) $\frac{e^{\sqrt{x}}}{2\sqrt{x}} + c$

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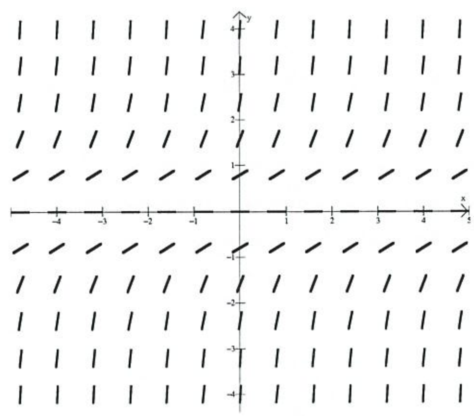
5. Which of the following differential equations corresponds to the slope field shown in the figure below?



// ROWS ∴ NO X
$x = 0 \Rightarrow \frac{dy}{dx} = 0$

a) $\frac{dy}{dx} = x^2 y$ ~~b)~~ $\frac{dy}{dx} = y^2$ ~~c)~~ $\frac{dy}{dx} = x^2$

d) $\frac{dy}{dx} = \frac{x^2}{y^2}$ e) $\frac{dy}{dx} = \frac{y}{x}$

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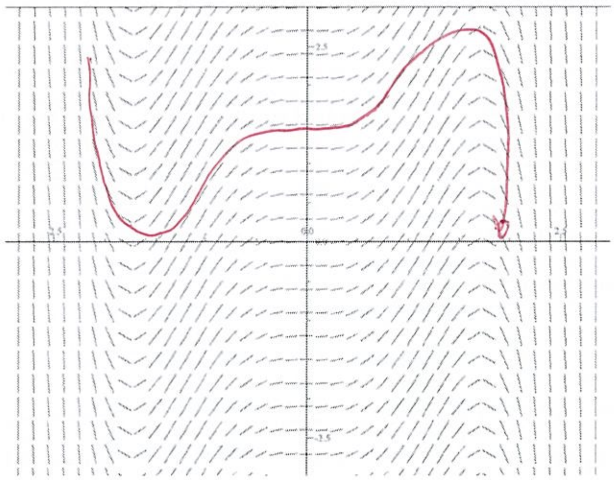
6. Which of the following equations might be the solution to the slope field shown in the figure below?



a) $y = 4x - x^3$ b) $y = x^3 - 4x$ c) $y = 4x^4 - x^6$

d) $y = x^3 - 15x^5$ e) $y = -\csc x$

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7. Identify is the mistake (if any) in this process:

$$\frac{dy}{dx} = 6x^2y^2$$

Step 1: $\frac{1}{y^2}dy = 6x^2dx$

Step 2: $\cancel{\ln|y^2|} = 2x^3 + c$

Step 3: $y^2 = e^{2x^3+c}$

Step 4: $y = \pm\sqrt{ke^{2x^3}}$

a) Step 1 b) Step 2 c) Step 3

d) Step 4 e) There is no mistake.

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8. $\displaystyle\int\left(\frac{z^4-6z-5}{3z^2}\right)dz = \int\left(\frac{1}{3}z^2 - 2z^{-1} - \frac{5}{3}z^{-2}\right)dz$

$$= \frac{1}{3}\frac{z^3}{3} - 2\ln|z| - \frac{5}{3}\frac{z^{-1}}{-1} + c$$

$$= \frac{1}{9}z^3 - 2\ln|z| + \frac{5}{3}z^{-1} + c$$

9. $\displaystyle\int\frac{x^5}{\left(x^2-1\right)^{5/2}}\,dx$ $\quad u = x^2 - 1 \quad x^2 = u+1$

$\quad du = 2x\,dx$

$$= \frac{1}{2}\int\frac{x^4\,(2x\,dx)}{(x^2-1)^{5/2}}$$

$$= \frac{1}{2}\int(u+1)^2\,u^{-5/2}\,du$$

$$= \frac{1}{2}\int(u^2+2u+1)\,u^{-5/2}\,du$$

$$= \frac{1}{2}\int\left(u^{-1/2} + 2u^{-3/2} + u^{-5/2}\right)du$$

$$= \frac{1}{2}\left[\frac{u^{1/2}}{1/2} + \frac{2u^{-1/2}}{-1/2} + \frac{u^{-3/2}}{-3/2}\right] + c$$

$$= u^{1/2} - 2u^{-1/2} \cancel{+} - \frac{1}{3}u^{-3/2} + c$$

$$= (x^2-1)^{1/2} + 2(x^2-1)^{-1/2} - \frac{1}{3}(x^2-1)^{-3/2} + C$$

10. $\int\left(7x^3 - x\cos^2\left(x^2\right) + \frac{\cos x}{e^{\sin x}}\right)dx$

$$= \int 7x^3\,dx - \frac{1}{2}\int \cos^2 x^2\,(2x\,dx) - \int e^{-\sin x}(-\cos x\,dx)$$

$$= \frac{7}{4}x^4 - \frac{1}{2}\left[\frac{1}{2}x^2 + \frac{1}{4}\sin 2x^2\right] - e^{-\sin x} + c$$

$$= \frac{7}{4}x^4 - \frac{1}{4}x^4 - \frac{1}{8}\sin 2x^2 - e^{-\sin x} + c$$

11. $\frac{1}{8}\int\left(8x\sqrt{4x^2+1}\right)dx$

$u = 4x^2+1$

$du = 8x\,dx$

$$= \frac{1}{8}\int u^{1/2}\,du$$

$$= \frac{1}{8}\,\frac{u^{3/2}}{3/2} + c = \frac{1}{12}\left(4x^2+1\right)^{3/2} + c$$

12. The acceleration of a particle is described by $a(t) = 36t^2 - 12t + 8$. Find the distance equation for $x(t)$ if $v(1) = 1$ and $x(1) = 3$.

$$v = \int(36t^2 - 12t + 8)\,dt$$

$$= 12t^3 - 6t^2 + 8t + C_1$$

$(1,1) \Rightarrow 1 = 12 - 6 + 8 + C_1 \rightarrow C_1 = -13$

$$x = \int(12t^3 - 6t^2 + 8t - 13)\,dt$$

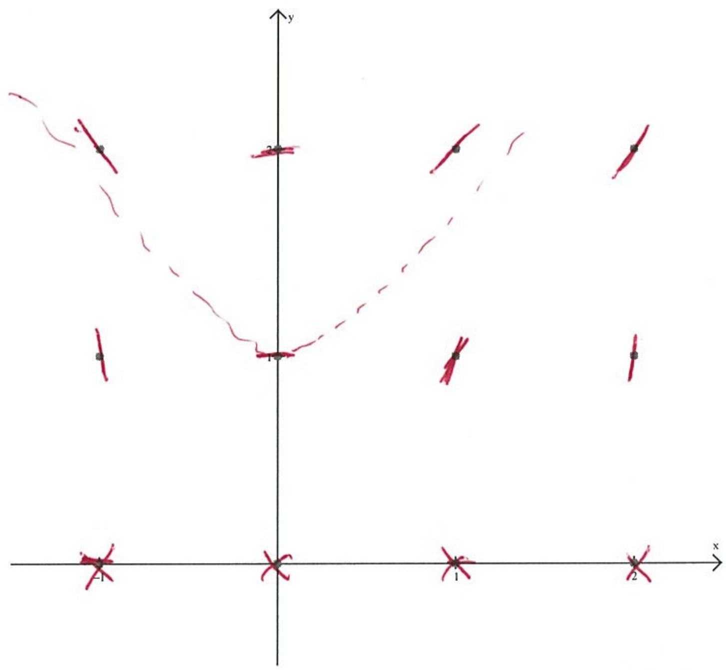
$$x = 3t^4 - 2t^3 + 4t^2 - 13t + C_2$$

$(1,3) \Rightarrow 3 = 3 - 2 + 4 - 13 + C_2 \Rightarrow C_2 = 11$

$$x(t) = 3t^4 - 2t^3 + 4t^2 - 13t + 11$$

13. Given the differential equation, $\frac{dy}{dx} = \frac{2x}{y}$

a. On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



b. If the solution curve passes through the point (0, 1), sketch the solution curve on the same set of axes as your slope field.

c. Find the equation for the solution curve of $\frac{dy}{dx} = x^2 y + x^2$ given that $y(3) = 0$

$= x^2(y+1)$

$\frac{1}{y+1}\,dy = x^2\,dx$

$\ln|y+1| = \frac{x^3}{3} + C$

$|y+1| = e^{x^3/3 + C} = k e^{\frac{x^3}{3}}$

$y(3)=0 \rightarrow 1 = k e^{9} \rightarrow k = e^{-9}$

$y+1 = e^{-9} e^{x^3/3}$

$y = -1 + e^{-9} e^{x^3/3}$