

Integral Test

Score _____

NO CALCULATOR ALLOWED

1. $\int_1^9 \frac{2}{\sqrt{x}} dx$

- a. 12 b. 4 c. 35 d. 36 e. 8

2. $\int_0^3 \sqrt{x^2 - 2x + 1} dx$

- a. 1 b.
- $\frac{3}{2}$
- c. 2 d.
- $\frac{5}{2}$
- e. 3

3. $\int_0^{1/3} \frac{9dt}{1+9t^2}$

- a. $\frac{\pi}{4}$ b. 0 c. $\frac{\pi}{2}$ d. $\frac{3\pi}{4}$ e. $\frac{5\pi}{4}$

4. The average value of e^{7x} on $x \in [0, 2]$ is

- a. $\frac{1}{14}e^{14}$ b. $\frac{1}{7}(e^{14} - 1)$ c. $\frac{1}{14}(e^{14} - 1)$
d. $\frac{1}{2}(e^{14} - 1)$ e. $\frac{1}{7}e^{14}$
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5. If $\int_0^k \frac{\sec^2 t}{1 + \tan t} dt = \ln 2$, then the value of k is

- a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{6}$ d. π e. $\frac{\pi}{4}$

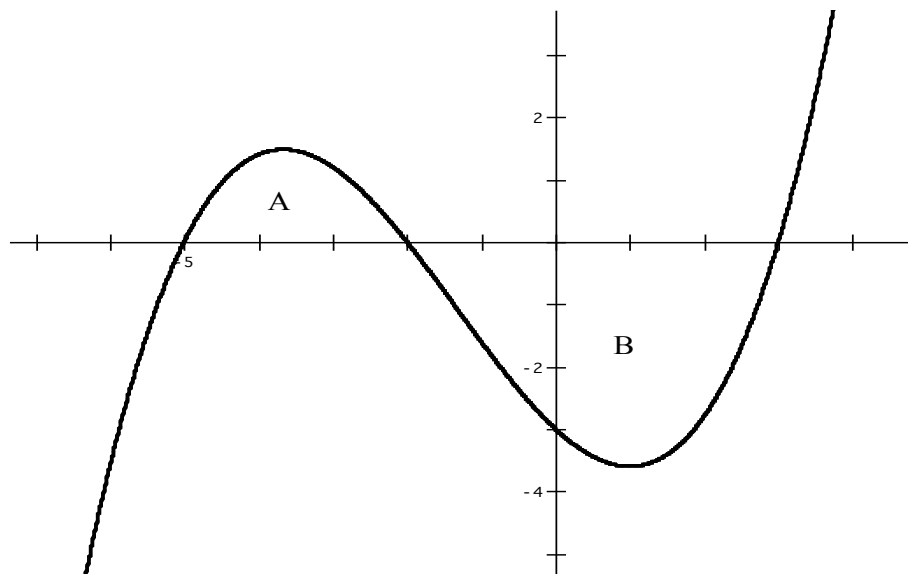
6. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	-0.1	1.1	1.4	1.2	1.5

If the Trapezoidal Rule is used to approximate $\int_1^5 f(x) dx$ the result is

- a. 4.2 b. 4.4 c. 4.6 d. 4.8 e. 5.0
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7. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x -axis.



In terms of A and B, $\int_{-5}^3 f(x) dx - 2 \int_{-2}^3 f(x) dx =$

- a. A b. $A - B$ c. $2A - B$ d. $A + B$ e. $A + 2B$

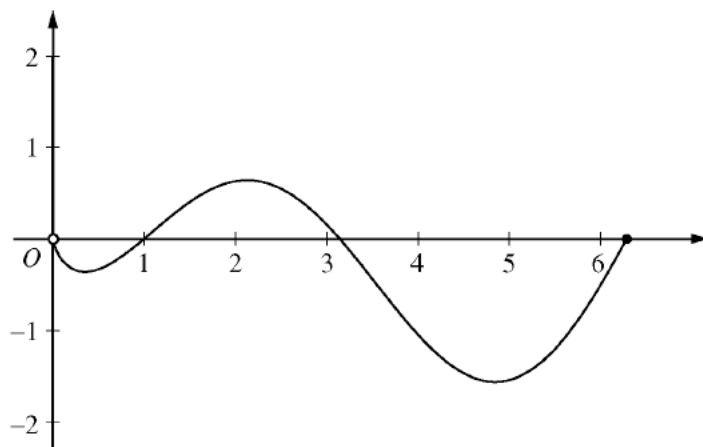
AP Calculus BC '15-16

Chapter 4 Definite Integrals

Calculator Required

Name:

1.



Graph of f

Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \leq 2\pi$.

The function g is defined by $g(x) = \int_1^x f(t) dt$ for $0 < x \leq 2\pi$.

a. Find $g(1)$ and $g'(1)$.

b. On what intervals, if any, is $g(x)$ increasing? Explain your reasoning.

c. For $x \in (0, 2\pi]$, find the value of x at which $g(x)$ has an absolute minimum. Justify your answer.

d. For $x \in (0, 2\pi]$, is there a value of x at which the graph of $g(x)$ is tangent to the x -axis? Explain your reasoning.

2. The following table lists known values of a twice differentiable function $f(x)$.

x	2	3	5	8	13
$f(x)$	0	4	-2	3	6

a. Estimate $f'(4)$

b. Use a left-hand Riemann sum to approximate $\int_2^{13} (f(x)) dx$.

c. Evaluate $\int_2^{13} (3 - 5f'(x)) dx$.

d. Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x \in [5, 8]$. Use the line tangent to $f(x)$ at $x = 5$ and the line secant to $f(x)$ on $x \in [5, 8]$ to show $\frac{4}{3} \leq f(7) \leq 4$.

3. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-2t} & \text{for } 5 < t \end{cases}$$

a. Is $r(t)$ continuous at $t = 5$? Show the work that leads to your answer.

b. Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

c. Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

d. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

End of
AP Calculus BC '15-16
Chapter 4
Definite Integrals