AP Calculus BC '16-17

Name Solution Key

Integral Test

Score____________

NO CALCULATOR ALLOWED

1. $\int_1^9 \frac{2}{\sqrt{x}}\, dx$ $= 2\frac{x^{1/2}}{1/2}\Big]_1^9 = 4(\sqrt{9}-1) =$

a. 12 b. 4 c. 35 d. 36 e. 8

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2. $\int_0^3 \sqrt{x^2-2x+1}\, dx$ $=$

a. 1 b. $\frac{3}{2}$ c. 2 d. $\frac{5}{2}$ e. 3

$= \int_0^3 \sqrt{(x-1)^2}\,dx = \int_0^3 (x-1)\,dx = \frac{x^2}{2} - x\Big]_0^3$

$= \frac{9}{2} - 3$

$u = 3t \quad du = \cancel{3dt}\ 3dt \qquad u(0) = 0$

$u(1/3) = 1$

3. $\displaystyle\int_0^{1/3} \frac{9dt}{1+9t^2} = 3\int_0^1 \frac{1}{u^2+1}\,du$

a. $\dfrac{\pi}{4}$ b. 0 c. $\dfrac{\pi}{2}$ d. $\dfrac{3\pi}{4}$ e. $\dfrac{5\pi}{4}$

$= 3\tan^{-1} u \Big]_0^1$

$= 3\left[\pi/4 - 0\right]$

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4. The average value of e^{7x} on $x \in [0, 2]$ is

a. $\dfrac{1}{14}e^{14}$ b. $\frac{1}{7}\left(e^{14}-1\right)$ c. $\frac{1}{14}\left(e^{14}-1\right)$

d. $\frac{1}{2}\left(e^{14}-1\right)$ e. $\frac{1}{7}e^{14}$

$\dfrac{1}{2-0}\displaystyle\int_0^2 e^{7x}\,dx = \frac{1}{2}\,\frac{e^{7x}}{7}\Big]_0^2$

$= \dfrac{1}{14}e^{14} - \dfrac{1}{14}e^0$

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5. If $\int_0^k \frac{\sec^2 t}{1+\tan t}\,dt = \ln 2$, then the value of k is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{6}$ d. π e. $\frac{\pi}{4}$

$u = 1 + \tan t$

$du = \sec^2 t\,dt$

$u(0) = 1$

$u(k) = 1 + \tan k$

$= \int_1^{1+\tan k} \frac{1}{u}\,du = \ln|u|\Big]_1^{1+\tan k}$

$= \ln|1+\tan k| = \ln 2$

$1 + \tan k = 2 \quad \tan k = 1$

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6. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	-0.1	1.1	1.4	1.2	1.5

If the Trapezoidal Rule is used to approximate $\int_1^5 f(x)\,dx$ the result is

a. 4.2 b. 4.4 c. 4.6 d. 4.8 e. 5.0

$\frac{1}{2}(-.1+1.1) + \frac{1}{2}(1.1+1.4) + \frac{1}{2}(1.4+1.2) + \frac{1}{2}(1.2+1.5)$

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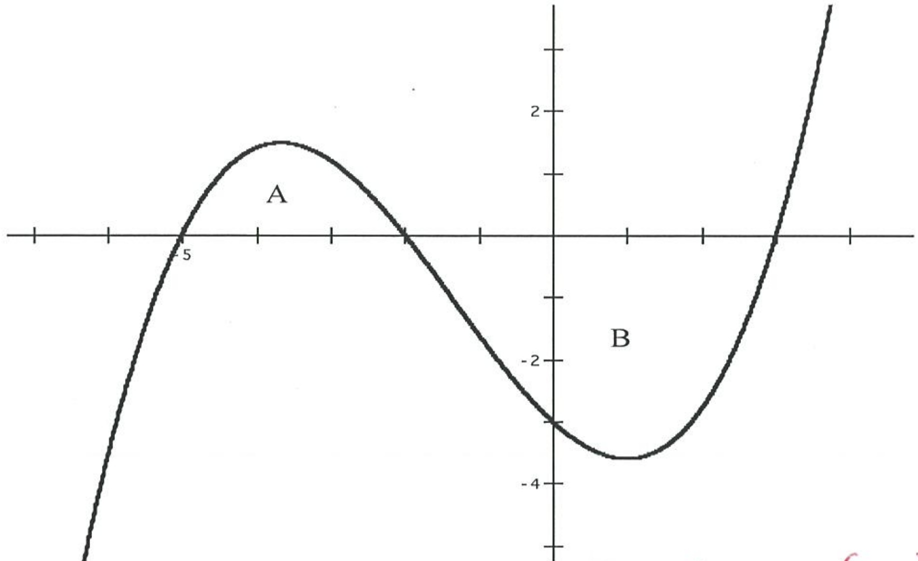
7. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x-axis.



In terms of A and B, $\int_{-5}^{3} f(x)\,dx - 2\int_{-2}^{3} f(x)\,dx =$ $(A - B) - 2(-B)$

a. A b. $A - B$ c. $2A - B$ d. $A + B$ e. $A + 2B$

$\int_{-5}^{-2} = A \quad \int_{-2}^{3} = -B$

AP Calculus BC '15-16

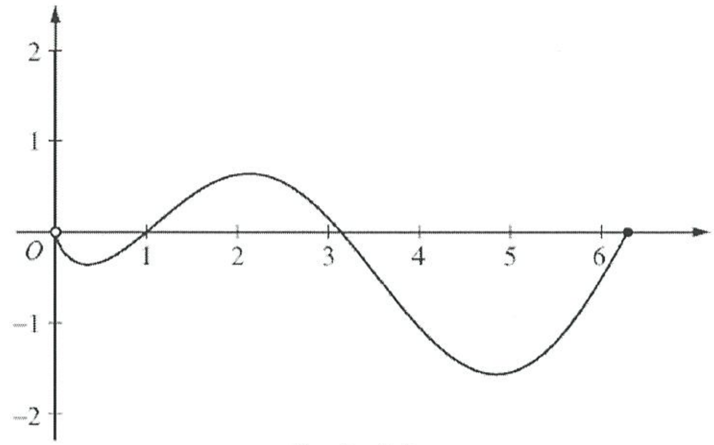
Chapter 4
Definite Integrals

Calculator Required

Name:

Solution Key

1.



Graph of f

Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$.

The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

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2 a. Find $g(1)$ and $g'(1)$.

$g(1) = \int_1^1 f(t)\,dt = 0$

$g'(1) = f(1) = 0$

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2 b. On what intervals, if any, is $g(x)$ increasing? Explain your reasoning.

$f = g'$ AND g IS INCREASING WHEN $g' \geq 0$

$x \in [1, \pi]$

3 c. For $x \in (0, 2\pi]$, find the value of x at which $g(x)$ has an absolute minimum. Justify your answer.

$g'(x) = f(x) = 0 \rightarrow x = 1, \pi, 2\pi$

x	g
1	0
π	.881
2π	-2.198

$g(\pi) = \int_1^{\pi} (\ln x)(\sin x) = .881$

$g(2\pi) = \int_1^{2\pi} \ln x \sin t = -2.198$

$\boxed{x = 2\pi}$

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2 d. For $x \in (0, 2\pi]$, is there a value of x at which the graph of $g(x)$ is tangent to the x-axis? Explain your reasoning.

$g(1) = 0$

$g'(1) = 0$

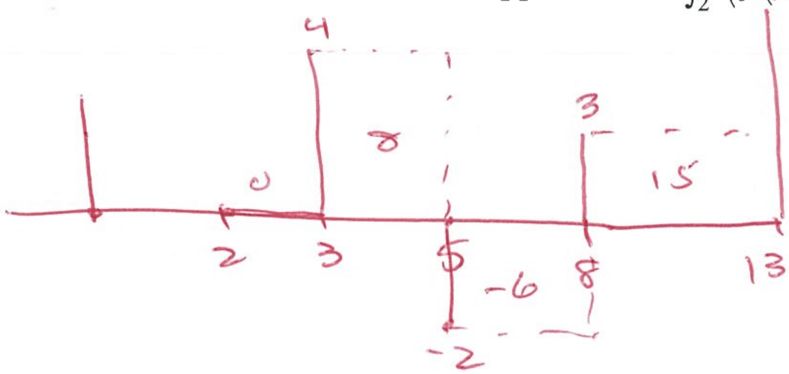
$\therefore$ yes. THERE IS A POINT WHERE THE X-AXIS IS THE TANGENT LINE

2. The following table lists known values of a twice differentiable function $f(x)$.

x	2	3	5	8	13
$f(x)$	0	4	-2	3	6

1 a. Estimate $f'(4) \approx \dfrac{f(5)-f(3)}{5-3} = \dfrac{-2-4}{2} = -3$

2 b. Use a left-hand Riemann sum to approximate $\int_2^{13} (f(x))\, dx$.



$0 + 8 + (-6) + 15$

$= 17$

3 c. Evaluate $\int_2^{13}(3-5f'(x))\,dx$.

$= 3\int_2^{13} dx - 5\int_2^{13} f'(x)\,dx$

$= 3x\Big]_2^{13} - 5\left[f(13) - f(2)\right]$

$= 3(13-2) - 5(6-0)$

$= 3$

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3 d. Suppose $f'(5)=3$ and $f''(x)<0$ for all $x\in[5,8]$. Use the line tangent to $f(x)$ at $x=5$ and the line secant to $f(x)$ on $x\in[5,8]$ to show $\frac{4}{3}\le f(7)\le 4$.

TANGENT LINE $y-(-2)=3(x-5) = 3x-17 = y_1$

$y_1(7) \approx 4$ AND 4 IS AN OVERESTIMATE BECAUSE f IS CONCAVE DOWN ~~CONCAVE DOWN~~ ON $[5,8]$

SECANT LINE: $m = \frac{3-(-2)}{8-5} = \frac{5}{3}$

$y_2-(-2) = \frac{5}{3}(x-5) \rightarrow y_2 = \frac{5}{3}x - \frac{19}{3}$

$y_2(7) = 4/3$

$\frac{4}{3} \le f(x) \le 4$ BECAUSE, SINCE $f''(x)<0$, f IS CONCAVE DOWN. THAT MEANS THE TANGENT LINE IS AN OVERESTIMATE AND THE SECANT IS AN UNDERESTIMATE.

3. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-.2t} & \text{for } 5 < t \end{cases}$$

2

a. Is $r(t)$ continuous at $t = 5$? Show the work that leads to your answer.

$$\frac{600(5)}{5+3} = 375$$

$$1000e^{-2(5)} = 367.879$$

$$\lim_{t \to 5^-} f(t) \neq \lim_{t \to 5^+} r(t)$$

3

b. Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

$$\frac{1}{8-0}\int_0^8 r(t)\,dt = \frac{1}{8}\left[\int_0^5 \frac{600t}{t+3}\,dt + \int_5^8 1000e^{-.2t}\,dt\right]$$

$$= 258.052$$

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(2) c. Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

$$\frac{d}{dt}\left(\frac{600t}{t+3}\right)\Bigg|_{t=3} = 50\ \frac{LT}{HR^2}$$

THE RATE AT WHICH THE TANK IS EMPTYING IS INCREASING BY 50 LT/HR/HR

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(2) d. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

$$\int_0^A r(t)\,dt = 3000$$

$$\text{OR} \quad 3000 = \int_0^5 \frac{600t}{t+3}\,dt + \int_5^A 1000e^{-.2t}\,dt$$

$$\int_5^A 1000e^{-.2t}\,dt = 1765.493$$