AP Calculus BC—Dr. Quatttrin

Derivative Test v8

Name Solution Key

Score____________

Directions: Show all work.

1. At what point on the graph of $y=\frac{1}{(x+1)^2}$ is the tangent parallel to the line $x-4y=-8$

$\frac{dy}{dx}=\frac{-2}{(x+1)^3}=\frac{1}{4}$

$-8=(x+1)^3$

$-2=x+1$

$-3=x$

(a) $\left(-6, \frac{1}{25}\right)$ (b) $\left(1, \frac{1}{4}\right)$ (c) $\left(-3, \frac{1}{4}\right)$

(d) $\left(-9, \frac{1}{64}\right)$ (e) $\left(3, \frac{1}{16}\right)$

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x)=f(g(x))$, $h'(8)=$

(a) -12 (b) -2 (c) -1 (d) -8 (e) 24

$h'(8)=f'(g(8))\cdot g'(8)$

$=f'(2)\cdot g'(8)=(-2)(4)=-8$

3. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$ with the initial condition $f(0) = 1$. What is the best approximation for $f(1)$ if Euler's method is used, starting at $x = 0$ with a step size of 0.5?

a) 1 b) 2 c) 2.5 d) $\sqrt{5}$ e) 3

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4. Which of the following statements must be true?

I. $\frac{d}{dx}\left(x \sec^{-1} x\right) = \sec^{-1} x + \frac{x}{\sqrt{x^2 - 1}}$

II. $\frac{d}{dx}\left(\frac{3 - 2x}{3x + 2}\right) = \frac{-13}{(3x+2)^2}$

III. $\frac{d}{dx} \ln(1 - x) = \frac{1}{1 - x}$

(a) I only (b) II only (c) III only

(d) II and III only (e) I and III

	M			
(0, 1)	0	y=1	.5	1
(.5, 1)	2	~~[illegible]~~ y−1=2(x−.5)	1	2

5. Assume the volume of a sphere is increasing at $12\ {}^{in^3}\!/_{sec}$. When the volume of the sphere is $36\pi\ in^3$, how fast is the surface area increasing?

a. 8 b. 8π c. 6

d. $\frac{8\pi}{3}$ e. 10

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$12 = 4\pi (3)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{3\pi}$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi (3)\left(\frac{1}{3\pi}\right)$

$= 8$

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6. The slope of the line tangent to $xy - y^3 + 6 = 0$ at (1,2) is

a. 0 b. $\frac{2}{11}$ c. $\frac{1}{6}$ d. $\frac{1}{4}$ e. $-\frac{1}{12}$

$x\frac{dy}{dx} + y(1) - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} + 2 - 12\frac{dy}{dx} = 0$

7. $\frac{d}{dx}\left[8x^3+7x^2-\frac{4}{3}x^{3/2}+e-\frac{5}{\sqrt[7]{x^5}}+\frac{2}{x}\right]$

$24x^3+14x-2x^{1/2}-\frac{25}{7}x^{-12/7}-\frac{2}{\cancel{x^2}}$

8. Let $y=e^{2\ln x}+\sin^{-1}(\cos 2x)$. Find y'.

$= x^2+\sin^{-1}(\cos 2x)$

$\frac{dy}{dx}=2x+\frac{1}{\sqrt{1 \cancel{+} \cos^2 2x)}}\cdot(-\sin 2x)\,(2)$

$=2x+\frac{1}{\sin 2x}\,(-2\sin 2x)$

$=2x-2$

9. If $4x^2 + 9y^2 = 36$, find $\frac{d^2y}{dx^2}$ in lowest terms.

$$8x + 18y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4x}{9y}$$

$$\frac{d^2y}{dx^2} = \frac{9y(-4) - (-4x)(9\frac{dy}{dx})}{(9y)^2}$$

$$= \frac{-36y + 36x\left(\frac{-4x}{9y}\right)}{81y^2} = \frac{-36y - \frac{16x^2}{y}}{81y^2}$$

$$= \frac{-(36y^2 + 16x^2)}{81y^3} = \frac{-4(36)}{81y^3}$$

$$= \frac{-16}{9y^3}$$

10. $f(x) = \ln(x^3 + 6x - 2)$; find $f''(x)$.

$$f'(x) = \frac{3x^2 + 6}{x^3 + 6x - 2}$$

$$f''(x) = \frac{(x^3+6x-2)(6x) - (3x^2+6)^2}{(x^3+6x-2)^2}$$

$$= \frac{6x^4 + 36x^2 - 12x - 9x^4 - 36x^2 - 36}{(x^3+6x-2)^2}$$

$$= \frac{-3x^4 \cancel{36x^3} - 12x - 36}{(x^3+6x-2)^2}$$

11. According to the adiabatic law for expansion of air, $P \cdot V^{7/5} = \frac{4}{81}$, where P is pressure and V is volume. At a specific instant, $P = 108$ lb/in^2 and is increasing at $\frac{dP}{dt} =$ 27 lb/in^2 per second. What is the rate of change of the volume at that moment?

$$\frac{d}{dt}\left[V^{7/5} = \frac{4}{81} P^{-1}\right]$$

$P = 108 \rightarrow V = \frac{1}{3}$

$$\frac{7}{5} V^{2/5} \frac{dV}{dt} = \frac{-4}{81} P^{-2} \frac{dP}{dt}$$

$$\frac{7}{5}\left(\frac{1}{3}\right)^{2/5} \frac{dV}{dt} = \frac{-4}{3}\left(\frac{1}{108^2}\right) = -1.143 \times 10^{-4}$$

$$\frac{dv}{dt} = -7.225 \times 10^{-3}$$