AP Calculus BC ’16-17

Fall Final
Part IIa

Calculator Required

Name:

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A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t = 0) and 8 P.M. (t = 8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(t), in hundreds of entries, at various times t are shown in the table above.

t hours	0	2	5	7	8
E(t) hundreds of entries	0	4	13	21	23

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a. Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time t = 6. Show the computations that lead to your answer.

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b. Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)dt$ in terms of the number of entries.

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c. t 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight (t = 12) ?

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d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

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2. A certain industrial chemical reaction produces synthetic oil at a rate of $S(t) = \frac{15t}{1+3t}$. At the same time, the oil is removed from the reaction vessel by a skimmer that has a rate of $R(t) = 2 + 5\sin\left(\frac{4\pi}{25}t\right)$. Both functions have units of gallons per hour, and the reaction runs from $t = 0$ to $t = 6$. At time of $t = 0$, the reaction vessel contains 2500 gallons of oil.

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a. How much oil will the skimmer remove from the reaction vessel in this six hour period? Indicate units of measure.

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b. Write an expression for $P(t)$, the total number of gallons of oil in the reaction vessel at time t.

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c. Find the rate at which the total amount of oil is changing at $t = 4$.

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d. For the interval indicated above, at what time t is the amount of oil in the reaction vessel at a minimum? What is the minimum value? Justify your answers.

3. The function f is defined as $f(x)=\begin{cases} 1+\sin x, & \text{if } x<0 \\ 1, & \text{if } x=0 \\ e^{-x}, & \text{if } 0<x \end{cases}$.

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a. Is $f(x)$ continuous at $x=0$? Justify your answer.

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b. Find $f'\left(-\frac{\pi}{2}\right)$ and $f'(\ln 3)$.

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c. Explain why $f'(0)$ does not exist.

d. Let $g(x)=\int_{-1}^{x} f(t)\,dt$. Find $g(1)$. Show the work that leads to your answer.

End of

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AP Calculus BC ’16-17

Fall Final
Part IIb

No Calculator Allowed

Name:

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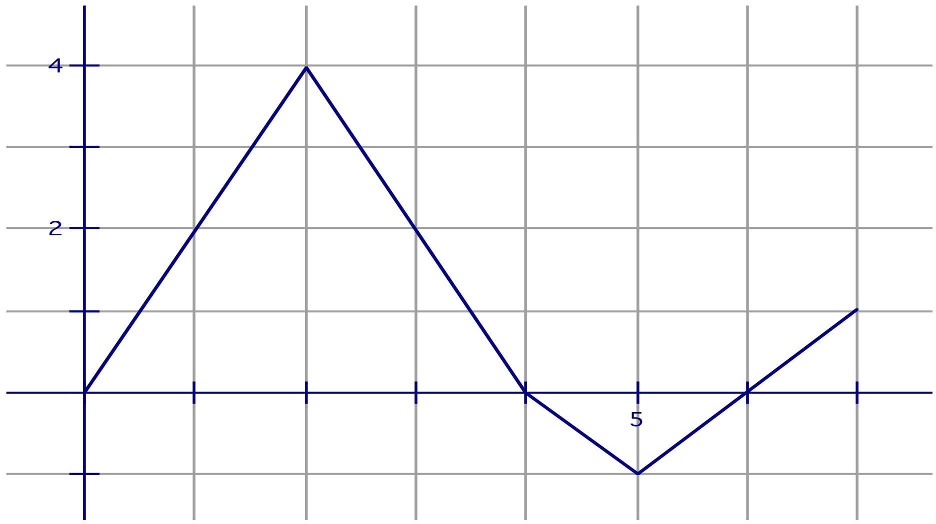
4. Let $g(x)=\int_2^x f(t)\,dt$ for $0 \le t \le 7$, where the graph of the differentiable function *f* is shown below.

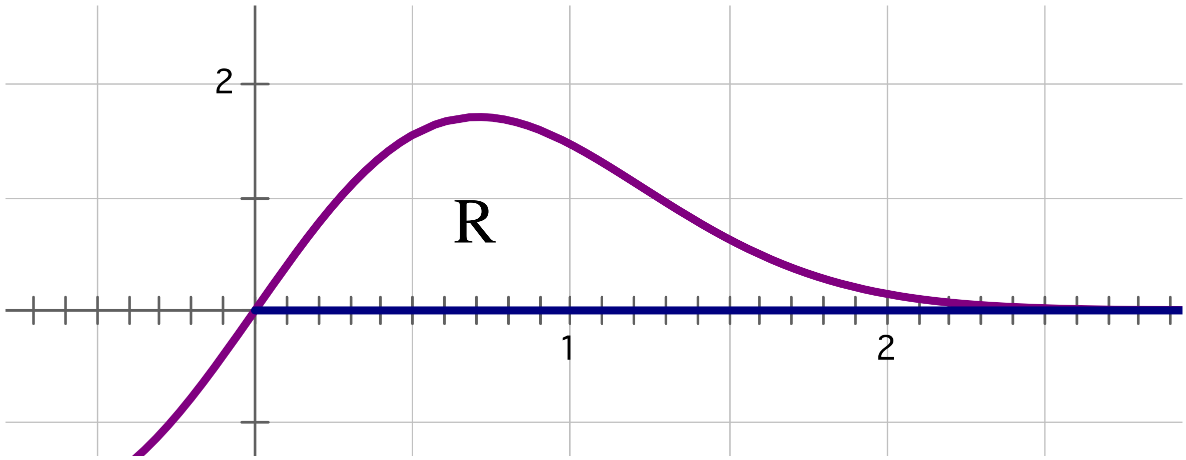


a. Find $g(3)$, $g'(3)$, and $g''(3)$.

b. Find the average rate of change of $g(x)$ on $0 \le c \le 3$?

c. For how many values of c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Justify your answer.

d. Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < t < 7$. Justify your answer.



5. The picture above is the graph of $f(x)=4xe^{-x^2}$.

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a. Find the area of the unbounded region in Quadrant I between $f(x)=4xe^{-x^2}$ and the x-axis.

b. The formula to find the volume of a solid formed by rotating a region about the x-axis is $V = \pi \int_a^b \left[f(x) \right]^2 dx$. Set up, but do not solve, the formula for the volume of the solid formed by revolving region R above about the x-axis.

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c. Show that $\sum_{n=0}^{\infty} a_n$ where $a_n = f(n)$ converges. Will the volume in part b converge? Why or why not?

6. Given the differential equation, $y' = y - 2x$

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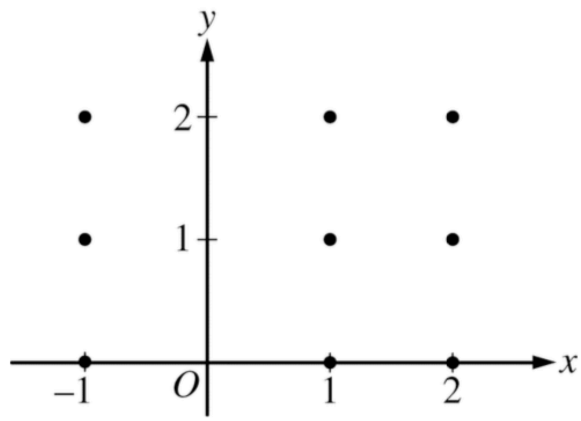
a) On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



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b) If the solution curve passes through the point (1, 0), sketch the solution curve on the same set of axes as your slope field.

c) Find the equation for the solution curve of $\frac{dy}{dx} = \frac{2x}{y}$, given that $y(0) = 1$

End of

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