BC Calculus '16-17
Integration Techniques Test

name SOLUTION KEY

Score___________

1. $\int \frac{dx}{x^2-2x-3}$ $\int \frac{A}{(x-3)} + \frac{B}{x+1}$

$A(x+1)+B(x-3)=1$

$x=-1$ $\quad -4B = 1$

$B = -1/4$

(a) $\frac{1}{4}\ln|(x-1)(x+3)|+c$

(b) $\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|+c$

(c) $\frac{1}{4}\ln\left|\frac{x+1}{x-3}\right|+c$

(d) $\frac{\ln|x^2-2x-3|}{2x-2}+c$

(e) $\frac{1}{2}\ln|x^2-2x-3|+c$

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2. $\frac{1}{2}\int_0^{\pi/6}\sec^2(2\theta)\tan(2\theta)\,2\,d\theta =$

$u = \tan 2\theta \quad du = \sec^2 2\theta \; 2\,d\theta$

a. $\frac{3}{4}$ b. $\frac{3}{2}$ c. $\sqrt{3}$ d. $\frac{2\pi}{3}$ e. $\pi\sqrt{3}$

$= \frac{1}{2}\int_0^{\sqrt{3}} u\,du = \frac{u^2}{4}\Big]_0^{\sqrt{3}} = \frac{3}{4}$

3. What is the best method to evaluate $\int \frac{5}{x^3 - 4x}\, dx = ?$

FACTORABLE DENOM

(a) Integration by Parts (b) Substitution (c) Partial Fractions

(d) Completing the Square (e) None of these

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4. $\frac{1}{4}\int \frac{5}{16x^2 - 7}\, 4dx =$ $u = 4x$ $a = \sqrt{7}$ $du = 4dx$ $\frac{1}{4}\int = \frac{1}{4}\left(\frac{1}{2\sqrt{7}} \ln\left|\frac{4x-\sqrt{7}}{4x+\sqrt{7}}\right|\right.$

(a) $\frac{5}{4\sqrt{7}} \tan^{-1}\left(\frac{4x}{\sqrt{7}}\right) + c$ (b) $\frac{5}{\sqrt{7}} \tan^{-1}\left(\frac{4x}{\sqrt{7}}\right) + c$

(c) $\frac{5}{\sqrt{7}} \ln\left|\frac{4x - \sqrt{7}}{4x + \sqrt{7}}\right| + c$ (d) $\frac{5}{2\sqrt{7}} \ln\left|\frac{4x - \sqrt{7}}{4x + \sqrt{7}}\right| + c$

(e) $\frac{5}{8\sqrt{7}} \ln\left|\frac{4x - \sqrt{7}}{4x + \sqrt{7}}\right| + c$

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5. The population of rabbits living on a farm grows logistically. The farmer counts 20 rabbits at time t=0 months, and notices that the population is growing fastest when there are 40 rabbits on the farm. Being a calculus whiz, the farmer estimates the carrying capacity of the farm to be:

(a) 40 rabbits
(b) 60 rabbits
(c) 80 rabbits
(d) 100 rabbits
(e) 800 rabbits

$40 = \frac{A}{2}$

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6. $\int_1^4 \ln\sqrt{y}\,dy =$

(a) $4\ln 2 - \frac{3}{2}$ (b) $\ln 2 - \frac{2}{3}$ (c) $4\ln 2 - \frac{14}{3}$

(d) $\ln 2 - \frac{5}{2}$ (e) none of these

$u = \ln\sqrt{y}$

$du = \frac{1}{\sqrt{y}}\frac{1}{2\sqrt{y}}\,dy$

$= \frac{1}{2y}\,dy$

$dv = dy$

$v = y$

$\int = y\ln\sqrt{y} - \int \frac{1}{2}\,dy$

$= y\ln\sqrt{y} - \frac{1}{2}y\Big]_1^4$

$= (4\ln 2 - 2) - (0 - \frac{1}{2})$

7. $\int e^{2\theta}\cos(4\theta)d\theta =$

$u = e^{2\theta}$ $dv = \cos 4\theta \, d\theta$

$du = 2e^{2\theta}$ $v = \frac{1}{4}\sin 4\theta \, d\theta$

$= e^{2\theta}\left(\frac{1}{4}\sin 4\theta \cancel{d\theta}\right) - \int \frac{1}{4}\sin 4\theta \left(2e^{2\theta}d\theta\right)$

$= \frac{1}{4}e^{2\theta}\sin 4\theta - \frac{1}{2}\int e^{2\theta}\sin 4\theta \, d\theta$

$u_2 = e^{2\theta}$ $dv_2 = \sin 4\theta$

$du_2 = 2e^{2\theta}d\theta$ $v_2 = -\frac{1}{4}\cos 4\theta \, d\theta$

$= \frac{1}{4}e^{2\theta}\sin 4\theta - \frac{1}{2}\left[e^{2\theta}\left(-\frac{1}{4}\cos 4\theta\right) - \int \frac{-1}{4}\cos 4\theta \left(2e^{2\theta}d\theta\right)\right]$

$\int e^{2\theta}\cos 4\theta \, d\theta = \frac{1}{4}e^{2\theta}\sin 4\theta + \frac{1}{8}e^{2\theta}\cos 4\theta - \frac{1}{4}\int e^{2\theta}\cos 4\theta$

$\frac{5}{4}\int e^{2\theta}\cos 4\theta \, d\theta = \frac{1}{4}e^{2\theta}\sin 4\theta + \frac{1}{8}e^{2\theta}\cos 4\theta + C$

$\int e^{2\theta}\cos 4\theta \, d\theta = \frac{1}{5}e^{2\theta}\sin 4\theta + \frac{1}{10}e^{2\theta}\cos 4\theta + C$

8. Find the volume of the solid whose base is bounded by $y = \sqrt{x \cos x}$, $y = 0$, $x = 0$, and $x = \dfrac{\pi}{2}$ and whose cross sections perpendicular to the x-axis are squares.

$$V = \int_0^{\pi/2} \left[\sqrt{x \cos x}\right]^2 dx$$

$$= \int_0^{\pi/2} x \cos x \, dx$$

$u = x \quad dv = \cos x \, dx$

$du = dx \quad v = \sin x$

$$= \left[x \sin x - \int \sin x \, dx \right]_0^{\pi/2}$$

$$= x \sin x + \cos x \Big]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} + 0\right) - (0 + 1)$$

$$= \frac{\pi}{2} - 1 = .571$$

9. $\displaystyle\int \frac{x^3-3x^2+9x-4}{x^2-2x+7}\,dx =$

$$\begin{array}{r} x - 1 \\ x^2-2x+7\,\overline{\big)\,x^3-3x^2+9x-4} \\ -(x^3-2x^2+7x) \\ \hline -x^2+2x-4 \\ (-x^2+2x-7) \\ \hline 3 \end{array}$$

$$= \int x - 1 + \frac{3}{x^2-2x+7}\,dx$$

$$= \int \left(x - 1 - \frac{3}{(x-1)^2+6} \right) dx$$

$$= \frac{x^2}{2} - x - \frac{3}{\sqrt{6}} \tan^{-1} \frac{x-1}{\sqrt{6}} + C$$