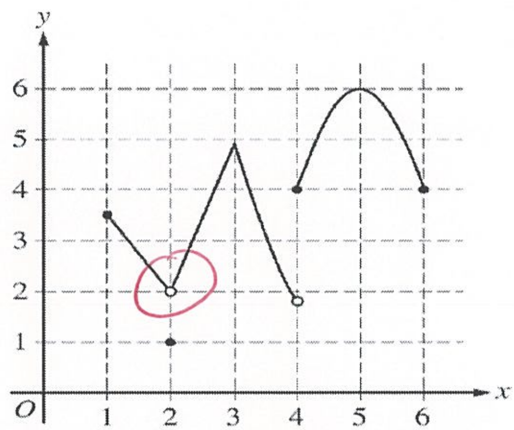
BC Calculus 2016-17
Limit and Continuity Test

Name Solution Key

Score______

1. The function f is defined on the interval $x \in [-5, 5]$ and has the graph shown below.



Graph of f

Which of the following is (are) true?

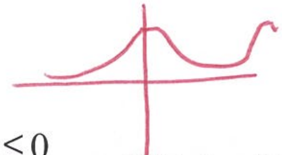
~~I.~~ $\lim_{x \to 2} f(x) = 1$ $= 2$

~~II.~~ $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = 5$ $\rightarrow$ DNE

III. $\lim_{x \to 3} f(x) = f(6) + 1$ $= 4 + 1 = 5$ YES

~~a)~~ I only b) II only c) III only

d) I and II only e) I and III only



2. Let $f(x)=\begin{cases} e^x, \text{ if } x\leq 0 \\ \cos x, \text{ if } 0<x \end{cases}$. Which of the following statements is true about f?

$e^0 = \cos 0 = 1$

$e^0 = 1 \neq -\sin 0$

I. f is continuous at $x = 0$.

~~II.~~ f is differentiable at $x = 0$.

III. f has a local maximum at $x = 0$. ~~Not Diff~~

~~a)~~ I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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3. The function f defined on all the Reals such that $f(x)=\begin{cases} x^2+kx-3 & \text{for } x\leq 1 \\ 3x+b & \text{for } x>1 \end{cases}$. For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(a) $k=-1,\ b=-3$
(b) $k=1,\ b=3$
~~(c)~~ $k=1,\ b=4$
(d) $k=1,\ b=-4$
~~(e)~~ $k=-1,\ b=6$

$1+k-3 = 3+b$

$2+k=3 \rightarrow k=1$

$f' = \begin{cases} \cancel{2x}+k \\ 3 \end{cases}$

$b=-4$

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4. $\lim_{h \to 0} \dfrac{\sin^2\left(\dfrac{\pi}{3}+h\right)-\dfrac{3}{4}}{h} =$ $\quad \dfrac{d}{dx}\sin^2 x\Big]_{x=\pi/3} = 2\sin x \cos x\Big|_{x=\pi/3}$

$= \sin 2x\Big|_{x=\frac{\pi}{3}}$

$= \sin\frac{2\pi}{3} = \frac{\sqrt{\ }}{2}$

(a) $\sqrt{3}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{2}$ (e) DNE

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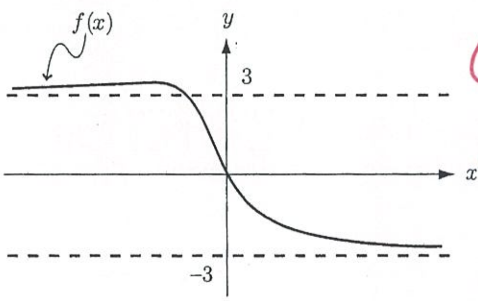
5. The figure below shows the graph of a function $f(x)$ which has horizontal asymptotes of $y=3$ and $y=-3$. Which of the following statements are true?



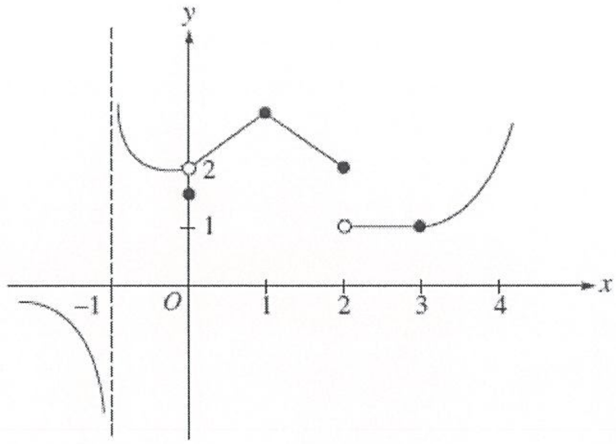
I. $f'(x)<0$ for all $x \geq 0$ f IS DECREASING

II. $\lim_{x\to-\infty} f(x)=3$ HA @ 3

III. $\lim_{x\to\infty} f'(x)=-3$ = SLOPE OF HA = 0

(a) I only (b) II only (c) III only (d) I and II only (e) I, II, and III

6. The function f is shown below. Which of the following statements about he function f, shown below, is false?



a) $\lim_{x \to 2} f(x)$ does not exist

b) $\lim_{x \to 3} f(x)$ exists

c) f is continuous at $x = 3$

d) $\lim_{x \to 3} \frac{f(x) - 5}{x - 3}$ exists $\nexists f(3)$ $\frac{1-5}{0}$

e) $\lim_{x \to 0} f(x)$ exists

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7. Which of the following improper integrals converge?

I. $\int_0^{\infty} \frac{1}{1+x^2}\, dx$ II. $\int_1^{\infty} \frac{1}{x^2}\, dx$ III. $\int_{-1}^{1} u^{-2} du$

(a) II only (b) I and II only (c) I and III only

(d) II and III only (e) I, II, and III

$$f(x)=\begin{cases} 3x-2, \text{ if } x<1 \\ \ln(3x-2), \text{ if } 1\le x \end{cases}$$

$f' = \begin{cases} 3 & x<1 \\ \dfrac{3}{3x-2} & x>1 \end{cases}$

8. Let $f(x)$ be the function defined above. Which of the following statements about $f(x)$ must be true?

~~I.~~ $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

II. $\lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^+} f'(x)$

~~III.~~ $f(x)$ is differentiable at $x=1$.

(a) None of these
(b) I only
(c) II only
(d) II and III only
(e) I, II, and III

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x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
3	0	0	7	5

9. Given that $f(x)$ is a thrice differentiable, continuous function on the interval $(0, 4)$ with the table values given above. $\lim\limits_{x\to 3}\dfrac{f(x)}{(x-3)^3} =$ $\dfrac{f'}{3(x-3)^2} = \dfrac{f''}{6(x-3)} = \dfrac{f'''}{6}$

$= \dfrac{7}{0}$

(a) 0 (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$ (e) dne

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10. $\lim_{x\to\infty} \tan^{-1}\left(\frac{x^{100}}{e^x}\right) =$ TAN^{-1}(0) =

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) 0 d) $-\frac{\pi}{4}$ e) Does not exist

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11. $\lim_{x\to\pi} \frac{\int_{\pi}^{x}(\cos^2 t)\,dt}{\sin 2x} =$ L'H $\frac{\cos^2 x}{\cos 2x(2)} = \frac{1}{2}$

a) -1 b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 1

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12. At $x=3$, the function given by $f(x)=\begin{cases} x^2, & \text{if } x<3 \\ 6x-9, & \text{if } 3\le x \end{cases}$ is $f' = \begin{cases} 2x \\ 6 \end{cases}$

CONT DIFF

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

AP Calculus BC '16-17
Limit/Continuity Test

Name SOLUTION KEY

Score__________.

1. $f(x)=\begin{cases} \cos x, \text{ if } x<0 \\ 1, \quad \text{if } x=0 \\ e^x, \text{ if } 0<x \end{cases}$

a) Is $f(x)$ continuous? Why/Why not?

i) $f(0)$ EXISTS? YES

ii) $\lim\limits_{x\to 0}$ EXISTS?

$$\lim_{x\to 0} \cos x = 1 = \lim_{x\to 0} e^x$$

iii) $f(0)=\lim\limits_{x\to 0} f(x)$

b) Is $f(x)$ differentiable? Why/Why not?

$$f'(x)=\begin{cases} -\sin x & \text{IF } x<0 \\ e^x & \text{IF } 0<x \end{cases}$$

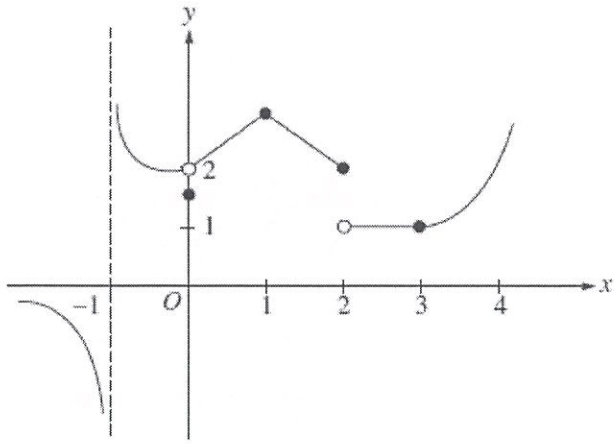
$\lim\limits_{x\to 1^-} f' = 0$ BUT $\lim\limits_{x\to 1^+} f' = 1$ $\therefore$ NOT DIFFERENTIABLE

2. Evaluate $\int_0^\infty \frac{1}{e^w}\,dw$

$$= \lim_{b\to\infty} \int_0^{b} e^{-w}\,dw = \lim_{b\to\infty} \left(-e^{-w}\right)\Big]_0^b$$

$$= \lim_{b\to\infty} -e^{-b} - \left(-e^{0}\right)$$

$$= 0+1 = 1$$



3. For this graph, find

(a) $\lim_{x\to -1^-} f(x) = \cancel{2}\ -\infty$ (b) $\lim_{x\to 2^-} f(x) = 2$ (c) $\lim_{x\to 0} f(x) = 2$

(d) $\lim_{x\to 2^+} f(x) = 1$ (e) $\lim_{x\to -1^+} f(x) = \infty$ (f) $\lim_{x\to 2} f(x) = \text{DNE}$

(g) $f(1) = 2$ (h) $\lim_{x\to 1^+} f(x) = 3$ (i) $f(2) = 2$ (j) $f(3) = 1$

4. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ $= \lim_{b\to 1^-} \int_0^b \frac{1}{\sqrt{1-x^2}}\,dx$

$$= \lim_{b\to 1^-} \sin^{-1} x \Big]_0^b$$

$$= \lim_{b\to 1^-} \sin^{-1} b - \sin^{-1} 0$$

$$= \frac{\pi}{2}$$