BC Calculus '16-17
Numerical Series Quiz

Name Solution Key

1. Determine the limit of the sequence $a_n = \dfrac{(n+1)!}{n!}$

a) 2 b) 3 c) $\frac{1}{3}$ d) $\frac{1}{2}$ e) Divergent

2. Which of the following sequences diverge?

I. $\left\{\dfrac{3n^5}{7n^4-1}\right\}$ II. $\left\{\dfrac{\cos n}{\pi^n}\right\}$ III. $\left\{\dfrac{n!}{e^{2n}}\right\}$

(A) I only (B) II only (C) I and III only

(D) II and III only (E) III only

3. Which of the following series diverge?

I. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ II. $\displaystyle\sum_{n=3}^{\infty}\frac{n^2}{2n^2+1}$ III. $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{2}\right)^n$

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I, II, and III

4. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+1}$ II. $\sum_{n=3}^{\infty} \frac{\pi^n}{3^n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(A) I only (B) II only (C) III only

(D) II and III only (E) I, II, and III

5. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$ II. $\sum_{n=2}^{\infty} \frac{2^n 3^{n+1}}{(2e)^n}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3+2}}$

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

6. Which of the following three tests will establish that the series $\sum_{n=1}^{\infty}\frac{3}{n(n+2)}$ converges?

I. Direct Comparison to $\sum_{n=1}^{\infty}3n^{-2}$

II. Limit Comparison to $\sum_{n=1}^{\infty}n^{-2}$

III. Direct Comparison to $\sum_{n=1}^{\infty}3n^{-1}$

$\frac{3}{n^2+2n} < \frac{3}{n^2}$ CONV

$\lim_{n\to\infty} \frac{3}{n(n+2)} \cdot \frac{n^2}{1} = 3$ CONV

$\frac{3}{n}$ DIV $\frac{3}{n(n+2)} < \frac{3}{n}$ INCONCLUSIVE

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

7. If $f(x)=\sum_{n=1}^{\infty}\left(\left(1-\cos x\right)^2\right)^n$, then $f\left(\frac{2\pi}{3}\right)=$

(A) 2 (B) $\frac{4}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) divergent

$= \sum_{n=1}^{\infty}\left(\left(1-\cos\frac{2\pi}{3}\right)^2\right)^n$

$= \sum_{n=1}^{\infty}\left(\left(1-\left(-\frac{1}{2}\right)\right)^2\right)^n$

$= \sum_{n=1}^{\infty}\left(\frac{9}{4}\right)^n$ DIV BECAUSE $r>1$

8. Which of the following series is/are absolutely convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ ABS II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CON III. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ NOT ALTERNATING

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I, II, and III

9. Which of the following series is/are convergent?

~~I.~~ $\sum_{n=1}^{\infty} \frac{1}{n^{-0.9}}$ $p<1$ II. $\sum_{n=1}^{\infty} \frac{3^n}{n+5^n} < \left(\frac{3}{5}\right)^n$ ~~III.~~ $\sum_{n=1}^{\infty} \frac{n}{1+4n}$ $\lim_{n\to\infty} a_n = 1/4$

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I, II, and III

10. For what values of k do both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n^2}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{3}\right)^n$ converges?

(A) 2
(B) 3
(C) 4
(D) 5
(E) All real values of k

$k < 3$

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Numerical Series Quiz

1. Use the Integral Test to determine if $\sum_{n=3}^{\infty} \frac{3n}{n^2-4}$ is convergent or divergent.

$$\int_3^{\infty} \frac{3x}{x^2-4}\,dx = \frac{3}{2}\int_3^{\infty} \frac{2x\,dx}{x^2-4}$$

$$= \frac{3}{2}\lim_{b\to\infty} \ln|x^2-4|\Big]_3^b$$

$$= \frac{3}{2}\lim_{b\to\infty} \ln|b^2-4| - 0$$

DNE

$\therefore \sum$ DIVERGES

2. Use one of the Comparison Tests to determine if $\sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3+n^2+3n}}$ is convergent or divergent.

COMP TO $\frac{3n}{\sqrt{n^3}} = \frac{3}{n^{1/2}}$ WHICH DIVERGES

$$\lim_{n\to\infty} \frac{3n}{\sqrt{n^3+n^2+3n}} \cdot \frac{n^{1/2}}{n^{1/2}} =$$

$$= \lim_{n\to\infty} \frac{n^{3/2}}{\sqrt{n^3+n^2+3n}} = 1 \quad \therefore \text{ BOTH DIVERGE}$$