AP Calculus BC ’16-17

Spring Final Part IIA

Calculator Allowed

Name:

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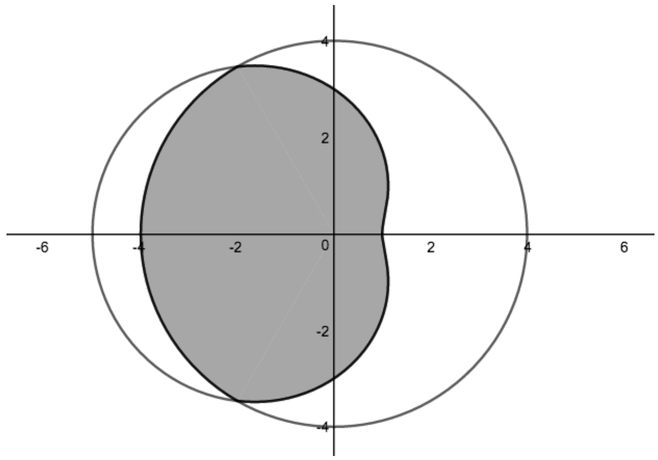
1. Letters arrive at a post office at a rate of $P(t) = 8 + t\sin\left(\frac{t^3}{80}\right)$ hundred letters per hour over the course of a workday. The day begins at 9am ($t = 0$) and ends at 5pm ($t = 8$). There are 300 letters in the office at 9am.

(a) Find $P'(2)$. Using correct units, interpret the meaning of $P'(2)$ in the context of this scenario.

(b) Find the total number of letters that arrive at the office between 9am and noon (t=3). Round to the nearest whole number of letters.

(c) What is the maximum number of letters in the office over the course of the workday ($0 \le t \le 8$)?



2. The polar graphs $r = 3 - 2\cos\theta$ and $r = 4$ are shown below.

(a) Find the exact values of points of intersection (r, θ) of the two curves.

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(b) Find the area of the shaded region.

(c) A particle travels along the curve $r = 3 - 2\cos\theta$ with time $t = \theta$. Write the particle's position vector $\langle x(t), y(t) \rangle$ and use it to find the speed of the particle when $t = 1$.

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NO Calculator Allowed

Name:

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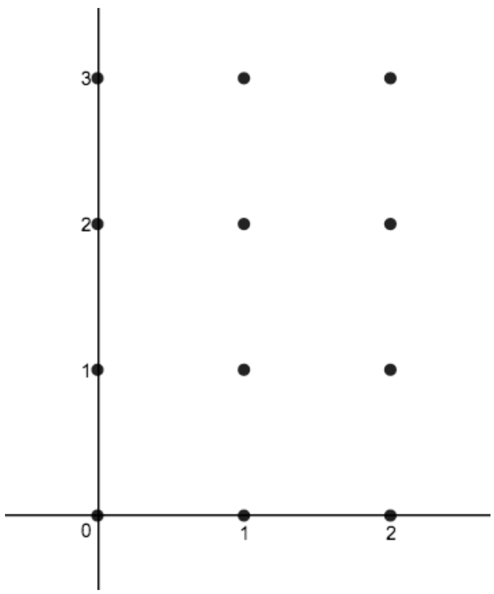
3. Suppose $\frac{dy}{dx} = (x^2 - 1)(2 - y)$

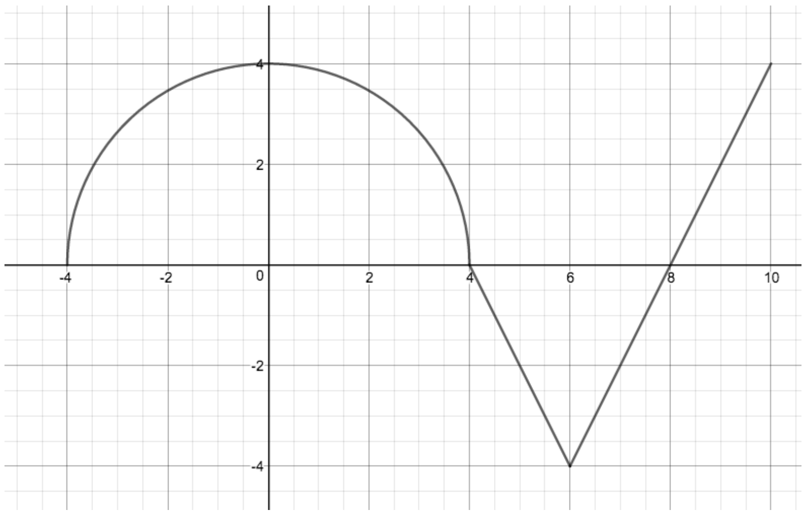
(a) Sketch the slope field on the 12 points indicated below.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. At the point $(1,3)$, does the graph of y have a maximum, minimum, or neither? Explain.

(c) Find the particular solution $y = f(x)$ that passes through the point $(1,3)$.

4. The graph of $g(x)$, defined on $x \in [-4, 10]$, is shown below. g(x) consists of a semi-circle and line segments. Let $f(x) = \int_6^x g(t)\,dt$.



graph of g

(a) Find $f(0)$ and $f(10)$.

(b) Write the equation of the line tangent to $f(x)$ at $x = 6$.

(c) On what interval(s), if any, is the graph of $f(x)$ both decreasing and concave up? Justify your answer.

(d) On $x \in [-4, 4]$, $g(x) = \sqrt{16 - x^2}$. Find the volume of the solid whose base is bounded above by g on $x \in [-4, 4]$ and below by the x-axis, and whose cross-sections perpendicular to the x-axis are squares.

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5. $f(x)$ is a twice-differentiable, strictly increasing function. Selected values of f and f' are given in the table below.

x	0	2	5	6	8
$f(x)$	-12	-5	0	1	7
$f'(x)$	1	4	1	2	3

(a) Approximate $f''(7)$.

(b) Must there be a value c at which $f''(c) = 0$? Explain.

(c) Use a right Riemann sum with 4 subintervals defined by the table to approximate $\int_0^8 f(x)\,dx$. Does this sum give an over- or under-approximation for $\int_0^8 f(x)\,dx$? Explain.

(d) Evaluate $\int_0^6 x\,f''(3x)\,dx$

6. Let f be a function with $f(1)=4$ whose derivatives at x=1 are given by $f^{(n)}(1)=\frac{(-1)^n}{2^n}(n-1)!$ for $n\geq 1$.

(a) Write the 3rd degree Taylor polynomial for f at x=1, $P_3(f,1)$.

(b) Show that $P_3(f,1)$ can be used to approximate $f(0.9)$ with an error of less than 0.00005.

(c) Find the radius and interval of convergence of the Taylor series for *f* at x=1.

(d) Write the first three terms and the general term for the Taylor series for $f'(x)$ at x=1.

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