AP Calculus BC '16-17 Name_____
Taylor Series Test

Directions: Show all work. Score_____.

1. What is the interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}(x-3)^n$?

(A) $(-1,1)$ (B) $[-1,1)$ (C) $(-1,1]$ (D) $(2,4]$ (E) $[2,4)$

---

2. If the Taylor series for *g(x)* about $x = 3$ is $g(x) = \sum_{n=0}^{\infty} \frac{x^n}{2n}$, then $g^{IV}(0)$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

3. The exact value of the series $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \cdots$ is

(A) 0 (B) -1 (C) e^{π} (D) $\ln \pi$ (E) undefined

---

4. Which of the following Series diverge?

I. $\sum_{n=1}^{\infty} 5^{-n} 6^{n-1}$ II. $\sum_{n=1}^{\infty} \frac{3n^5}{7n^4 - 1}$ III. $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(A) I only (B) III only (C) I and II only
(D) II and III only (E) I, II, and III

---

5. What are the first three non-zero terms of the power series for xe^{-x} ?

(A) $x - x^2 - \frac{x^3}{2}$ (B) $x - x^2 + \frac{x^3}{2}$ (C) $-x + x^2 - \frac{x^3}{2}$
(D) $x + x^2 + \frac{x^3}{2}$ (E) $1 - x - \frac{x^2}{2}$

---

6. The Taylor series for $f(x)$ is $1 + x + x^2 + \cdots + x^n + \cdots$, and the Taylor series for $g(x)$ is $1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots$. Find the 3rd degree Taylor polynomial for the product $f(x) \cdot g(x)$.

(A) $1 - x^3$ (B) $1 + x$ (C) $1 + x + x^2 + x^3$ (D) $1 + x + x^3$ (E) $1 + x - x^2$

---

7. $\sum_{n=1}^{\infty}(-1)^n x^{3n+1}$ is the Maclaurin series for which function?

(A) $\frac{1}{1+x^{3x+1}}$ (B) xe^{x^3} (C) $\frac{x}{1+x^3}$ (D) $x!e^{3x+1}$ (E) $3x+1$

---

8. Which of these is the Maclaurin series for $\cos\left(\sqrt{x}\right)$?

(A) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!}$ (B) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ (C) $\sum_{n=0}^{\infty}\frac{x^n}{2n!}$

(D) $\sum_{n=0}^{\infty}\frac{x^{2n-1}}{(2n-1)!}$ (E) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(2n)!}$

---

9. The radius of convergence for $\sum_{n=0}^{\infty}\frac{2^n}{3^n n^2}x^n$ is

(A) $2/3$ (B) $3/2$ (C) 3 (D) 0 (E) ∞

---

1. Let $f(x) = \sin\left(x^2\right) + \cos x$

a. Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x = 0$.

b. Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$.

c. Use the series found in part (a) and part (b) to write the first four nonzero terms of the Taylor series for $f(x)$ about $x = 0$.

d. Find the value of $f^{VI}(0)$.

2. The Maclaurin series for the function $f(x)$ is given by $\sum_{n=0}^{\infty} \frac{(-1)^n (n+1) x^{3n}}{n!}$ on its interval of convergence. The first four terms of the Maclaurin series for the function $f(x)$ is $M_9(f, 0) = 1 - 2x^3 + \frac{3}{2!}x^6 - \frac{4}{3!}x^9$.

a. Find the interval of convergence for the Maclaurin series of $f(x)$. Justify your answer.

b. Find the approximate values of $f(.5)$. What is the maximum difference between the exact value of $f(.5)$ and the approximate value of $f(.5)$ found in part.

c. What is the maximum error between the exact value of $f(.5)$ and the approximate value of $f(.5)$ found in part b.

d. Let $h(x)=x^2 f'(x)$. Find the first three non-zero terms and the general terms of $h(x)$.