BC Calculus '16-17
Volume Test

name Solution Key

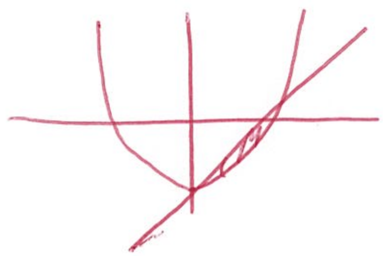
Score______________

1. The area of the region enclosed by $y = x^2 - 4$ and $y = x - 4$ is given by

(a) $\int_0^1 (x - x^2)\,dx$ (b) $\int_0^1 (x^2 - x)\,dx$ (c) $\int_0^2 (x - x^2)\,dx$

(d) $\int_0^2 (x^2 - x)\,dx$ (e) $\int_0^4 (x^2 - x)\,dx$

$x^2 - 4 = x - 4$
$x^2 = x$
$x = 0, 1$



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2. Which of the following integrals gives the length of the graph $y = \tan x$ between $x=a$ to $x=b$ if $0 < a < b < \frac{\pi}{2}$?

(a) $\int_a^b \sqrt{x^2 + \tan^2 x}\,dx$ (b) $\int_a^b \sqrt{x + \tan x}\,dx$

(c) $\int_a^b \sqrt{1 + \sec^2 x}\,dx$ (d) $\int_a^b \sqrt{1 + \tan^2 x}\,dx$

(e) $\int_a^b \sqrt{1 + \sec^4 x}\,dx$

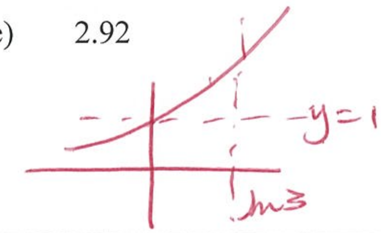
$\frac{dy}{dx} = \sec^2 x$

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

3. Let R be the region in the first quadrant bounded by $y = e^{x/2}$, $y = 1$ and $x = \ln 3$. What is the volume of the solid generated when R is rotated about the x-axis?

(a) 2.80 (b) 2.83 (c) 2.86 (d) 2.89 (e) 2.92

$V = \pi \int_0^{\ln 3} (e^{x/2})^2 - 1^2 \, dx$



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4. A region is bounded by $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. A solid is formed by revolving the region about the x-axis. The volume of the solid

(a) is independent of m.

(b) increases as m increases.

(c) decreases as m increases.

(d) increases until $m = \frac{1}{2}$, then decreases.

(e) is none of the above

$V = \pi \int_m^{2m} \frac{1}{x^2} \, dx$

$= \pi \left. \frac{-1}{x} \right]_m^{2m}$

$= \pi \left[\frac{-1}{2m} - \frac{-1}{m} \right]$

$= \pi \left(\frac{1}{2m} \right)$

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5. Let R be the region in the first quadrant bounded by $y = \sin^{-1} x$, the y-axis, and $y = \frac{\pi}{2}$. Which of the following integrals gives the volume of the solid generated when R is rotated about the y-axis?

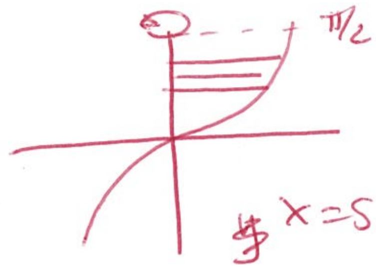
(a) $\pi \int_0^{\pi/2} y^2 \, dy$

(b) $\pi \int_0^1 \left(\sin^{-1} x\right)^2 dx$

(c) $\pi \int_0^{\pi/2} \left(\sin^{-1} x\right)^2 dx$

(d) $\pi \int_0^{\pi/2} \left(\sin y\right)^2 dy$

(e) $\pi \int_0^1 \left(\sin y\right)^2 dy$



$x = \sin y$

DISK METHOD

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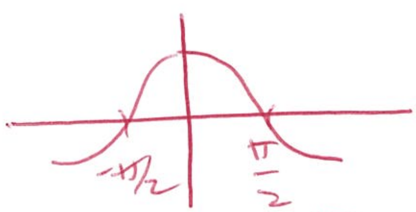
6. The base of a solid is the region enclosed by $y = \cos x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. If each cross-section of the solid perpendicular to the x-axis is a square, the volume of the solid is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi^2}{2}$ (e) 2

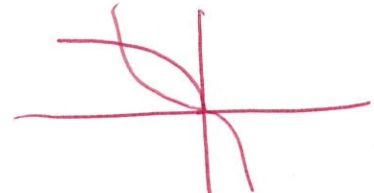


$A = s^2 = \cos^2 x$

$$V = \int_{-\pi/2}^{\pi/2} \cos^2 x \, dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x \Big]_{-\pi/2}^{\pi/2}$$

$$= \left(\frac{\pi}{4} + 0\right) - \left(-\frac{\pi}{4} + 0\right) = \pi/2$$



7. Let T be the region bounded by $y = -x^3$ and $y = \sqrt{-8x}$.

a) Find the volume of the solid generated when T is rotated about the x – axis. Show the anti-differentiation steps.

WASHER WITH VERTICAL RECTANGLES

$$V = \pi \int_{-1.516}^{0} (\sqrt{-8x})^2 - (-x^3)^2 \, dx$$

$$= \pi \int_{-1.516}^{0} \left[(-8x) - x^6\right] dx$$

$$= \pi \left[-4x - \frac{x^7}{7}\right]_{-1.516}^{0} = 20.621$$

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b) Find the volume of the solid generated when T is rotated about the y – axis. Show the anti-differentiation steps.

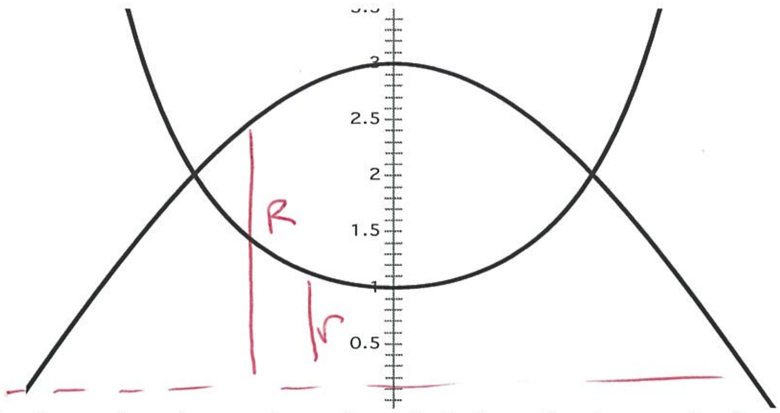
WASHER METHOD WITH HORIZONTAL RECTANGLES

$$x = \frac{y^2}{8} \qquad x = (-y)^{1/3}$$

$$V = \pi \int_{0}^{3.482} \left(\frac{y^2}{8}\right)^2 - \left[(-y)^{1/3}\right]^2 dy$$

$$= \pi \int_{0}^{3.482} \left(\frac{y^4}{64} - y^{2/3}\right) dy$$

$$= \pi \left[\frac{y^5}{320} - \frac{y^{5/3}}{5/3}\right]_{0}^{3.482} = 10.053$$



8. Let S be the region shown above bounded above by the graph of $y = 3\cos x$ and below the graph of $y = e^{x^2}$.

a) Find the volume of the solid generated when S is revolved about the x-axis.

$$V = \pi \int_{-.83596017}^{.83596017} (3\cos x)^2 - (e^{x^2})^2 \, dx$$

$$= 28.515$$

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b) Let the base of the solid be the region S. Find the volume of the solid where the cross-sections perpendicular to the x-axis are rectangles which are twice as tall as they are wide.

$$V = \int_{-.835}^{.835} 2(3\cos x - e^{x^2})^2 \, dx = \cancel{3.712} \quad 7.425$$

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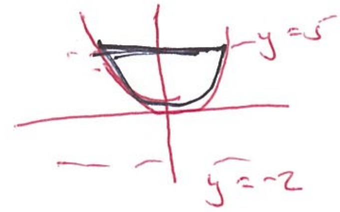
9. Let R be the region bounded by $y = x\tan^{-1}\left(\frac{x}{5}\right)$, and ~~$x=5$~~ $y=5$.



a) Find the volume of the solid generated when R is rotated about the line $y=-2$.

$$V = \pi \int_{-5.8116992}^{5.8116992} 7^2 - \left(x \tan^{-1}\frac{x}{5} + 2\right)^2 \, dx$$

$$= 1161.720$$

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b) Find $\frac{dy}{dx}$.

$$= x \frac{1}{\left(\frac{x}{5}\right)^2 + 1}\left(\frac{1}{5}\right) + \tan^{-1}\frac{x}{5}$$

$$= \frac{5x}{x^2+25} + \tan^{-1}\frac{x}{5}$$

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c) Find the perimeter of region R.

$$P = 11.6233984 + \int_{-5.812}^{5.812} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$= 11.623 + 15.793$$

$$= 27.416$$