BC Calculus '17-18
Integration Techniques Test

name SOLUTION KEY

Score_____

1. $\int \frac{4}{x^2-4x-12}\,dx =$

$(x-6)(x+2)$

$\frac{A}{x-6} + \frac{B}{x+2} = 4$

$A(x+2)+B(x-6)=4$

$x=-2$ $\quad B=-1/2$

$x=6$ $\quad A=1/2$

(a) $\frac{1}{2}\ln\left|\frac{x+2}{x-6}\right|+c$

(b) $\frac{1}{2}\ln\left|\frac{x-6}{x+2}\right|+C$

(c) $\frac{1}{8}\ln\left|(x-6)(x+2)\right|+C$

(d) $\frac{1}{8}\ln\left|(x-6)(x+2)\right|+C$

(e) $\frac{1}{8}\ln\left|\frac{x-6}{x+2}\right|+C$

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2. $\int xe^{2x}\,dx =$

(a) $\frac{1}{4}e^{2x}(2x-1)+c$ (b) $\frac{1}{2}e^{2x}(2x-1)+c$ (c) $\frac{1}{4}e^{2x}(4x-1)+c$

(d) $\frac{1}{2}e^{2x}(x-1)+c$ (e) $\frac{1}{4}e^{2x}(x-1)+c$

$u=x \qquad dv=e^{2x}dx$

$du=dx \qquad v=\frac{1}{2}e^{2x}$

$uv-\int v\,du = x\left(\frac{1}{2}e^{2x}\right)-\int \frac{1}{2}e^{2x}dx$

$= \frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+C$

$= \frac{1}{4}e^{2x}(2x-1)$

3. What is the best method to evaluate $\int \frac{1}{x^2+4x+7}\,dx = ?$

(a) Integration by Parts (b) Substitution (c) Partial Fractions

(d) Completing the Square (e) None of these

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4. $\int \frac{e^{x^2}-2x}{e^{x^2}}dx =$ $\int 1\,dx + \int -2xe^{-x^2}\,dx$ $u = e^{-x^2}$ $du = -2x\,dx$

$x + e^{-x^2} + c$

a) $-e^{-x^2}+c$
b) $-e^{x^2}+c$
c) $x-e^{x^2}+c$
d) $x+e^{-x^2}+c$
e) $x-e^{-x^2}+c$

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5. What is the best method to evaluate $\int \frac{dx}{x\sqrt{4x^2-9}}$? SEC^{-1} FORMULA

a) Integration by Parts b) Substitution c) Partial Fractions

d) Completing the Square e) None of these

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6. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{4000}P(400-P)$, where $P(0) = 100$. What is the **maximum rate of change** of $P(t)$?

MAXIMUM RATE OCCURS AT $P = \frac{1}{2}A$

$= 200$

$\left.\frac{dP}{dt}\right|_{P=200} = \frac{1}{4000}(200)(400-200) = 10$

a) 10

b) 100

c) 200

d) 400

e) 4000

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7. Which of the following statements are true?

I. $\int (\sin^3 x \cos^2 x)\,dx = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + c$

II. $\int \sec 2x\,dx = 2\sec 2x \tan 2x + c$ THIS IS THE DERIVATIVE, NOT $\int$

III. $\int \left(\frac{3x^2+6x-4}{(x^3+3x^2-4x+2)^2} \right) dx = \ln\left|x^3+3x^2-4x+2\right|^2 + c$

$\int = \int \frac{1}{u^2} \neq \ln u$

a) I only b) II only c) III only

d) I and II only e) II and III only

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7. Find the volume of the solid formed when the region bounded by $y = x^2e^{-2x}$ and the x-axis on $x \in [-1, 0]$ is revolved about the x-axis. Show the anti-differentiation.

$$V = \pi \int_{-1}^{0} (x^2 e^{-2x})^2 \, dx$$

$$= \pi \int x^4 e^{-4x} \, dx$$

$dv = e^{-4x}$

$u_1 = x^4$ $\quad v_1 = \frac{-1}{4} e^{-4x}$

$u_2 = 4x^3$ $\quad v_2 = \frac{1}{16} e^{-4x}$

$u_3 = 12x^2$ $\quad v_3 = \frac{-1}{64} e^{-4x}$

$u_4 = 24x$ $\quad v_4 = \frac{+1}{256} e^{-4x}$

$u_5 = 24$ $\quad v_5 = \frac{-1}{1024} e^{-4x}$

$$= \pi \left[\frac{-1}{4} x^4 e^{-4x} - \frac{1}{4} x^3 e^{-4x} + \left(\frac{-3}{16}\right) e^{-4x} - \frac{3}{32} e^{-4x} + \frac{-1}{64} e^{-4x} \right]_{-1}^{0}$$

$$= 20.027$$

8. $\int x \cot^{-1} x^2 \, dx$

$w = x^2$

$dw = 2x\,dx$

$= \frac{1}{2} \int \cot^{-1} w \, dw$

$u = \cot^{-1} w \qquad dv = dw$

$du = \frac{-1}{w^2+1} \qquad v = w$

$= \frac{1}{2}\left[w \cot^{-1} w - \int \frac{-w}{w^2+1} \, dw \right]$

$= \frac{1}{2}\left[w \cot^{-1} w + \frac{1}{2} \int \frac{2w}{w^2+1} \, dw \right]$

$= \frac{1}{2} w \cot^{-1} w + \frac{1}{4} \ln(w^2+1) + C$

$= \frac{1}{2} x^2 \cot^{-1} x^2 + \frac{1}{4} \ln(x^4+1) + C$

9. $\int \frac{2x^4+3x^3-14x^2-7x+18}{x^3-7x+6}\,dx$

SCRATCH

$$\begin{array}{r} 2x+3 \\ x^3-7x+6\overline{)\,2x^4+3x^3-14x^2-7x+18} \\ -(2x^4 \quad -14x^2+12x) \\ \hline 3x^3 \quad -19x+18 \\ -(3x^3 \quad -21x+18) \\ \hline 2x \end{array}$$

$$= \int 2x+3+\frac{2x}{(x+3)(x-1)(x-2)}$$

1	1	0	−7	6
		1	1	6
	1	1	−6	0

$(x-1)(x^2+x-6)$

$$= \int 2x+3+\frac{A}{x+3}+\frac{B}{x-1}+\frac{C}{x-2}$$

$A(x-1)(x-2)+B(x+3)(x-2)+C(x+3)(x-1)=2x$

$x=-3$: $20A = -6$

$x=1$: $-4B = +2$

$x=2$: $5C = 4$

$C = \frac{4}{5}$

$$= \int \left(2x+3+\frac{-\frac{3}{10}}{x+3}+\frac{-\frac{1}{2}}{x-1}+\frac{\frac{4}{5}}{x-2}\right)dx$$

$$= x^2+3x-\frac{3}{10}\ln|x+3|-\frac{1}{2}\ln|x-1|+\frac{4}{5}\ln|x-2|+C$$