AP Calculus BC '17-18
Anti Derivative Test

Name SOLUTION KEY

Score___________

1. Which of the following statements are true?

I. $\int \frac{1}{x\sqrt{x^2-16}}\,dx = \sec^{-1}\frac{x}{4} + c$ NO $= \frac{1}{4}\sec^{-1}\frac{x}{4} + c$

II. $\int \csc x\,dx = \ln|\csc x - \cot x| + c$ YES

III. $\int \left(e^{6x}\sin e^{6x}\right)dx = \frac{1}{6}\cos e^{6x} + c$ NO $= -\frac{1}{6}\cos e^{6x} + c$

a) I only b) II only c) III only

d) I and II only e) II and III only

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2. $\int x\sqrt{1-x^2}\,dx$ $u = 1-x^2 \quad du = -2x\,dx$

a) $\frac{\left(1-x^2\right)^{3/2}}{3} + c$ b) $-\left(1-x^2\right)^{3/2} + c$ c) $\frac{x^2\left(1-x^2\right)^{3/2}}{3} + c$

d) $\frac{-x^2\left(1-x^2\right)^{3/2}}{3} + c$ e) $\frac{-\left(1-x^2\right)^{3/2}}{3} + c$

$$\int x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int u^{1/2}\,du$$

$$= -\frac{1}{2}\,\frac{u^{3/2}}{3/2} + c$$

$u = \ln x \quad du = \frac{1}{x} dx$

3. $\int \frac{1+\ln x}{x \ln x} \, dx =$ $= \int \frac{1+u}{u} du = \int (\frac{1}{u} + 1) du = \ln|u| + u + c$

a) $\frac{1}{2}(1+\ln x)^2 + c$

b) $x + \ln x + c$

c) $\ln(\ln x) + c$

d) $\ln x + \ln(\ln x) + c$

e) $\frac{\ln x}{x + \ln x} + c$

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4. $\int (x^3)\sqrt{1-x^2} \, dx$

$u = 1 - x^2 \quad du = -2x dx$
$x^2 = 1 - u$

a) $\frac{x^4}{2} \cdot \frac{(1-x^2)^{3/2}}{3} + c$

b) $-\frac{1}{2}(1-x^2)^{1/2} - \frac{1}{3}(1-x^2)^{3/2} + c$

c) $-\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + c$

d) $\frac{1}{3}(1-x^2)^{3/2} - \frac{1}{5}(1-x^2)^{5/2} + c$

e.) $\frac{-(1-x^2)^{3/2}}{3} + c$

$\int \frac{-1}{2} \int (1-u) u^{1/2} du$

$= -\frac{1}{2} \int (u^{1/2} - u^{3/2}) + c$

$= -\frac{1}{2} \left[\frac{u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2} \right] + c$

$= -\frac{1}{3} u^{3/2} + \frac{1}{5} u^{5/2} + c$

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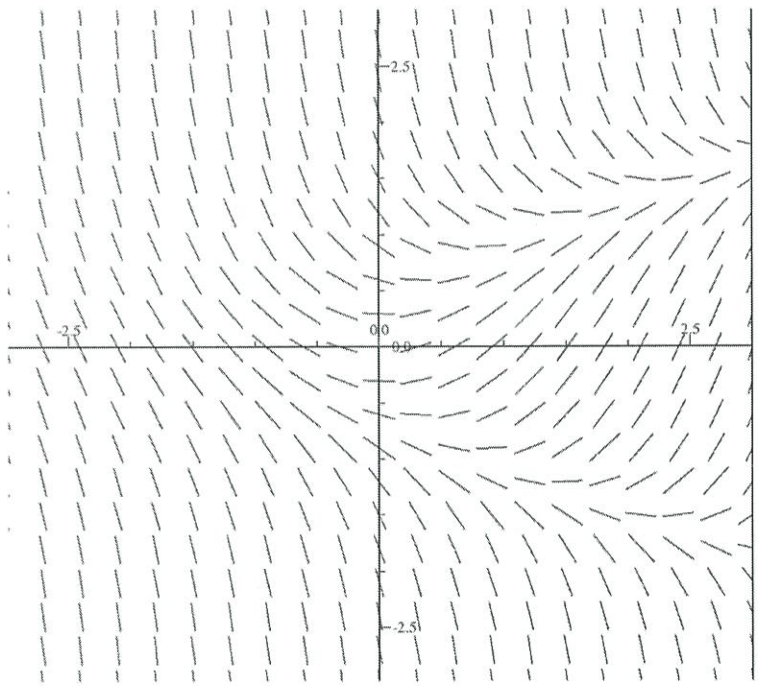
5. Which of the following differential equations corresponds to the slope field shown in the figure below?



a. $\frac{dy}{dx} = x - y^2$

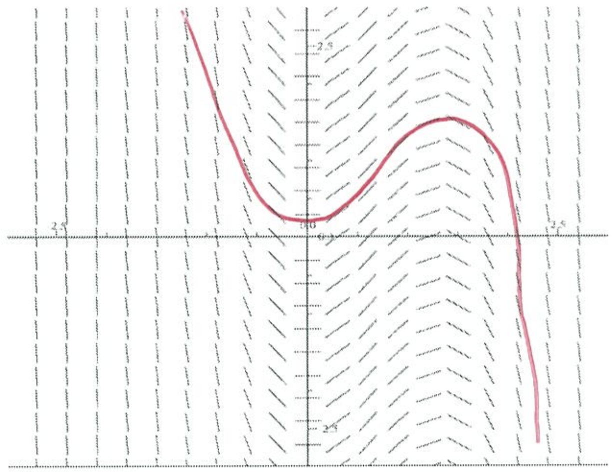
~~b.~~ $\frac{dy}{dx} = 1 - \frac{y}{x}$

~~c.~~ $\frac{dy}{dx} = -y^3$

~~d.~~ $\frac{dy}{dx} = x - \frac{1}{2}x^3$

~~e.~~ $\frac{dy}{dx} = x + y$

6. Which of the following equations might be the solution to the slope field shown in the figure below?



a) $y=4x-x^3$ b) $y=x^3-4x$ c) $y=4x^4-x^6$

d) $y=x^3-15x^5$ e) $y=\sec x$

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7. Identify is the first mistake (if any) in this process:

$$\frac{dy}{dx}=xy+x$$

Step 1: $\frac{1}{y+1}dy=xdx$

Step 2: $\ln|y+1|=x^2+c$ SHOULD BE $\frac{1}{2}x^2+c$

Step 3: $|y+1|=e^{x^2}+c$

Step 4: $y=e^{x^2}+c$

a) Step 1 b) Step 2 c) Step 3

d) Step 4 e) There is no mistake.

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8. $\int \left(\frac{t^3-4t-3}{5t^{2/3}}\right) dt$

$= \int \left(\frac{1}{5} t^{\cancel{8}\,7/3} - \frac{4}{5}t^{1/3} - \frac{3}{5}t^{-2/3}\right) dt$

$= \frac{1}{5}\cdot\frac{3}{\cancel{10}} t^{28/3} - \frac{4}{5}\cdot\frac{3}{4}t^{4/3} - \frac{3}{5}\left(\frac{3}{1}\right)t^{1/3} + c$

$= \frac{3}{\cancel{1}50} t^{\cancel{28}/3} - \frac{3}{5}t^{4/3} - \frac{9}{5}t^{1/3} + c$

9. $\frac{1}{3}\int \frac{3x^5}{(x^3-1)^{3/2}} dx$ $\quad u = x^3 - 1 \rightarrow x^3 = u+1$

$du = 3x^2 du$

$\frac{1}{3}\int (u+1)u^{-3/2} du$

$= \frac{1}{3}\int u^{-1/2} + u^{-3/2}\, du$

$= \frac{1}{3}\left[\frac{u^{1/2}}{1/2} + \frac{u^{-1/2}}{-1/2}\right] + c$

$= \frac{2}{3}(x^3-1)^{1/2} - \frac{2}{3}(x^3-1)^{-1/2} + c$

10. $\int\left(3x^8+\frac{\sec^2 x}{e^{\tan x}}-x^3\cot\left(x^4\right)\right)dx$

$= \int 3x^8\,dx + \int \sec^2 x\, e^{-\tan x}\,dx - \frac{1}{4}\int 4x^3 \cot x^4\,dx$

$= \frac{1}{3}x^9 + \left(-e^{-\tan x}\right) - \frac{1}{4}\ln\left|\sin x^4\right| + C$

11. The acceleration of a particle is described by $a(t)=48t^2-18t+6$. Find the distance equation for $x(t)$ if $v(1)=1$ and $x(1)=3$.

$v(t)=\int(48t^2-18t+6)\,dt$

$= 16t^3-9t^2+6t+C_1 \rightarrow (1,1) \Rightarrow C=-12$

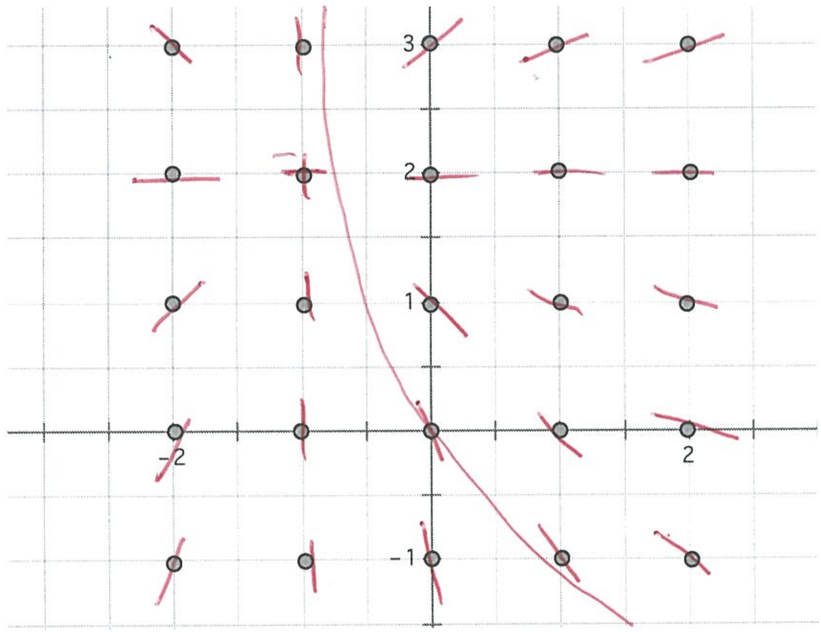
$x(t)=\int(16t^3-9t^2+6t-12)\,dt$

$= 4t^4-3t^3+3t^2-12t+C_2 \quad (1,3)\Rightarrow C_2=11$

$x(t)=4t^2-3t^3+3t^2-12t+11$

12. Given the differential equation, $\frac{dy}{dx} = \frac{y-2}{x+1}$

a. On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



b. If the solution curve passes through the point (0, 0), sketch the solution curve on the same set of axes as your slope field.

c. Find the equation for the solution curve of $\frac{dy}{dx}=(y-2)(x+1)$ given that $y(0)=5$

$$\frac{1}{y-2}\,dy=(x+1)\,dx$$

$$\ln|y-2|=\frac{x^2}{2}+x+C$$

$$|y-2|=e^{x^2/2+x+C}=ke^{\frac{x^2}{2}+x}$$

$(0,5)\Rightarrow 3=ke^0$

$3=k$

$$y-2=3e^{\frac{x^2}{2}+x}$$

$$y=2+3e^{\frac{x^2}{2}+x}$$