AP Calculus BC—Dr. Quattrin
Derivative Test v9

Name Solution Key

Score____________

Directions: Show all work.

1. Let f be a differentiable function with $f(2)=3$ and $f'(2)=-5$, and let g be a function defined by $g(x)=xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x=2$?

a) $y=3x$

b) $y-3=-5(x-2)$

c) $y-6=-5(x-2)$

d) $y-6=-7(x-2)$

e) $y-6=-10(x-2)$

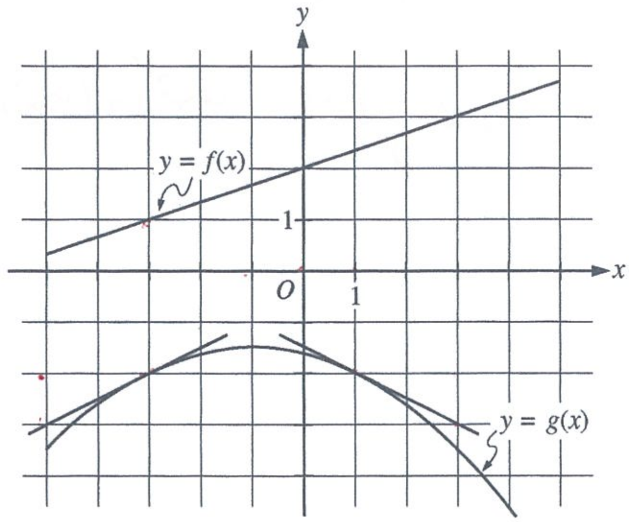
$g' = xf' + f(1)$

$g(2) = 2(3) = 6$

$g'(2) = 2(-5) + 3(1) = -7$

$y - 6 = -7(x-2)$

2. The figure below shows the graph of the functions f and g. The graphs of the lines tangent to the graph of g at $x=-3$ and $x=1$ are also shown. If $B(x)=g(f(x))$, what is $B'(-3)$?



a) $-\frac{1}{2}$

b) $-\frac{1}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{3}$

e) $\frac{1}{2}$

$B' = g'(f(-3))\cdot f'(-3)$

$= g'(1)\cdot f'(-3) = -\frac{1}{2}\left(\frac{1}{3}\right) = -1/6$

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3. Which of the following statements must be true?

I. $\frac{d}{dx}\left(x\csc^{-1}x\right)=\csc^{-1}x-\frac{1}{\sqrt{x^2-1}}$

II. $\frac{d}{dx}\left(\frac{3-2x}{3x+2}\right)=\frac{-13}{(3x+2)^2}$

~~III~~. $\frac{d}{dx}\ln(1-x)=\frac{1}{1-x}$

a) I only

~~b)~~ II only

~~c)~~ II and III only

d) I and II only

e) I and III

I) $x\left(\frac{1}{\sqrt{x^2-1}}\right)+\csc^{-1}x\,(1)$

III $\frac{1}{1-x}(-1)=\frac{1}{x-1}$

II $=\frac{(3x+2)(-2)-(3-2x)(3)}{(3x+2)^2}=\frac{-13}{(3x+2)^2}$

4. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y + xy$ with the initial condition $f(0) = 1$. What is the best approximation for $f(2)$ if Euler's method is used, starting at $x = 0$ with a step size of 1?

a) 0 b) 1 c) 2 d) 6 e) 24

(x, y)	m	TANGENT LINE	NEW x	NEW y
(0, 1)	1	$y - 1 = 1(x - 0)$	1	2
(1, 2)	4	$y - 2 = 4(x - 1)$	2	6

$\frac{d}{dx}(f(x^2)) = f'(x^2)\,(2x)$

$= g(x^2)\,(2x)$

5. If $\frac{d}{dx}[f(x)] = g(x)$ and $\frac{d}{dx}[g(x)] = f(3x)$, then $\frac{d^2}{dx^2}[f(x^2)]$ is

a) $4x^2 f(3x^2) + 2g(x^2)$ b) $f(3x^2)$ c) $f(x^4)$

d) $2xf(3x^2) + 2g(x^2)$ e) $2xf(3x^2)$

$\frac{d^2}{dx^2} = \frac{d}{dx}\, g(x^2)[2x] = 2x\, g'(x^2)(2x) + 2g(x^2)$

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6. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

a) $-\frac{3}{2}$ b) $-\frac{3}{4}$ c) 0 d) $\frac{3}{4}$ e) $\frac{3}{2}$

$2y\frac{dy}{dx} + 3(xy+1)^2\left[x\frac{dy}{dx} + y(1)\right] = 0 \rightarrow -2\frac{dy}{dx} + 3\left(2\frac{dy}{dx} + (-1)\right) = 0$

$4\frac{dy}{dx} = 3$

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7. $\frac{d}{dx}\left[-8x^7 + 7x - \frac{4}{3}x^{5/3} - \frac{5}{\sqrt[10]{x^3}} + \frac{1}{17x}\right] = \frac{d}{dx}\left[-8x^7 + 7x^1 - \frac{4}{3}x^{5/3} - 5x^{-3/10} + \frac{1}{17}x^{-1}\right]$

$= -56x^6 + 7 - \frac{20}{9}x^{2/3} + \frac{3}{2}x^{-13/10} - \frac{1}{17}x^{-2}$

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8. If $g(x)=\sin^{-1}x^2$, find $g''(x)$

$$g'(x)=\frac{1}{\sqrt{1-x^4}}(2x)$$

$$g''(x)=\frac{(1-x^4)^{1/2}(2)-\cancel{2}x\left(\frac{1}{\cancel{2}}(1-x^4)^{-1/2}(-4x^3)\right)}{1-x^4}$$

$$=\frac{2(1-x^4)^{1/2}+\dfrac{4x^4}{(1-x^4)^{1/2}}}{1-x^4}$$

$$=\frac{2(1-x^4)+4x^4}{(1-x^4)^{3/2}}=\frac{2(1+x^4)}{(1-x^4)^{3/2}}$$

9. If $f(x)=e^{\tan x}$, find $f''(x)$.

$$f'(x)=e^{\tan x}\sec^2 x$$

$$f''(x)=e^{\tan x}(2\sec x\sec x\tan x)+\sec^2 x\, e^{\tan x}\sec^2 x$$

$$=\sec^2 x\, e^{\tan x}\left[2\tan x+\sec^2 x\right]$$

10. Given $\frac{d}{dx}\left[y^2+2x=x^2+2xy+1.\right] \Rightarrow 2y\frac{dy}{dx}+2=2x+2x\frac{dy}{dx}+2y(1)$

a) Show that $\frac{dy}{dx}=\frac{x+y-1}{y-x}$

$$(2y-2x)\frac{dy}{dx}=2x+2y-2$$

$$\frac{dy}{dx}=\frac{2x+2y-2}{2y-2x}=\frac{x+y-1}{y-x}$$

b) Find the slopes of the tangent lines at all the points where the curve intersects the line $y=2$.

$y=2 \Rightarrow 4+2x=x^2+4x+1$

$0=x^2+2x-3=(x+3)(x-1)$

$x=1,-3$ $\qquad \left.\frac{dy}{dx}\right|_{(1,2)}=2 \qquad \left.\frac{dy}{dx}\right|_{(-3,2)}=\frac{-2}{5}$

c) Determine all the points on the curve which had a horizontal tangent line.

HORIZ TAN: $\frac{dy}{dx}=0 \Rightarrow x+y-1=0 \Rightarrow y=1-x$

ON THE CURVE MEANS $\quad (1-x)^2+2x=x^2+2x(1-x)+1$

$1-\cancel{2x}+x^2+\cancel{2x}=x^2+2x-2x^2+1$

$0=-2x^2+2x=-2x(x-1)$

$x=0,1$

$x=0 \Rightarrow y=\pm 1 \quad (0,1)$ HAS $m=0$ BUT $(0,-1)$ DOES NOT

$x=1 \Rightarrow y=0$ OR $2 \quad (1,0)$ HAS $m=0$ BUT $(1,2)$ DOES NOT

$\boxed{(0,1),(1,0)}$