BC Calculus '17-18
Dx Apps Test
No Calculator

Name Solution Key

1. If $g'(x) = 3xe^{2x} + 5$, then $g(x)$ has a point of inflection at:

a) $x = -1$ b) $x = \frac{1}{2}$ c) $x = -\frac{1}{2}$

d) $x = 1$ e) Nowhere

$g''(x) = 3xe^{2x}(2) + e^{2x}(3)$
$= 3e^{2x}(2x+1) = 0$

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	0	-2	8	0
4	8	0	0	3
8	0	-12	0	4

Then at $x = 8$, $g(x)$ has a:

a) Relative Maximum b) Relative Minimum

c) Point of Inflection d) Zero

e) None of these

$g' = 0$ & $g'' > 0$

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3. Suppose $f'(x)=\dfrac{(x+1)^3(x-4)^2}{(x^4+1)}$. Which of the following statements must be true?

I. The slope of the line tangent to $y=f(x)$ at $x=1$ is 36.

II. $f(x)$ is increasing on $x\in(1,\ 4)$

III. $f(x)$ has a minimum at $x=-1$

f' − 0 +

−1

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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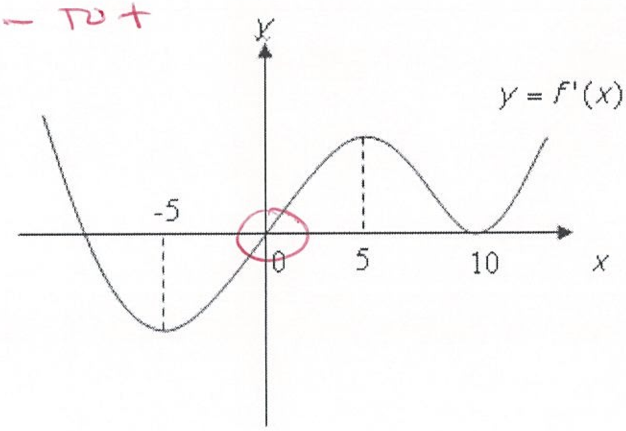
4. Below is the graph of $f'(x)$. For what value(s) of x does $f(x)$ have a minimum?

f' SWITCHES − TO +



a) 0 only

b) 0 and 10

c) -5 and 5

d) -5 and 10

e) None of these

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5. A particle moves along the y-axis with position $s(t)=15t^4-20t^3$. On what interval(s) is the particle slowing down?

a) $t\in\left[\tfrac{2}{3},\infty\right)$

b) $t\in(-\infty,1]$

c) $t\in(-\infty,0]\cup\left[\tfrac{2}{3},1\right]$

d) $t\in(-\infty,0]\cup\left[1,\tfrac{2}{3}\right]$

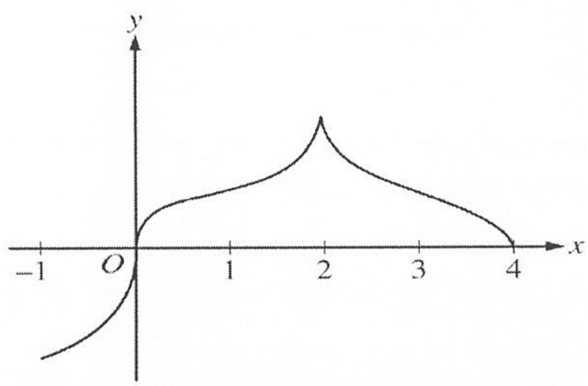
e) $t\in(-\infty,0]$

$v = 60t^3 - 60t^2$
$= 60t^2(t-1)$

v: − 0 − 0 +
t: 0, 1

$a = 180t^2 - 120t$
$= 60t(3t-2)$

a: + 0 − 0 + 0 +
t: 0, 2/3, 3/2



Graph of f'

6. The graph of f' is shown above. The line tangent to f' at $x=0$ is vertical, and f' is not differentiable at $x=2$. Which of the following statements is TRUE?

a.) f' does not exist at $x=2$.

b) f is decreasing on the interval (2, 4)

c) The graph of f has a point of inflection at $x=0$

d) The graph of f has a relative maximum at $x=0$

e) The graph of f has a point of inflection at $x=2$

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7. A particle's acceleration function is $a(t)=\sqrt[7]{7t-1}$, and its velocity is 0 at $t=0$. Which of these represents the particle's velocity function?

a) $v(t)=\frac{1}{8}\left[(7t-1)^{8/7}-1\right]$

b) $v(t)=\frac{1}{8}\left[7(5t-1)^{8/7}+1\right]$

c) $v(t)=\frac{1}{8}\left[7(8t-1)^{8/7}-1\right]$

d) $v(t)=\frac{1}{7}(7t-1)^{-6/7}$

e) $v(t)=\frac{1}{7}\left[(7t-1)^{-6/7}-1\right]$

$v=\int (7t-1)^{1/7}\,dt$

$=\frac{1}{7}\frac{(7t-1)^{8/7}}{8/7}+C$

$=\frac{1}{8}(7t-1)^{8/7}+C$

$(0,0)\Rightarrow 0=\frac{1}{8}(-1)^{8/7}+C$

$C=-1/8$

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x	-2	0	3	5	6
$f'(x)$	3	1	4	7	5

8. Let f be a polynomial function with values $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

a) The graph of f is concave up.

b) The graph of f has at least two points of inflection.

c) f is increasing.

d) f has no critical points.

e) f has at least two relative extrema.

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9. Find the absolute minimum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval $[0, 2]$.

$f' = 6x^2 + 6x - 12 = 6(x+2)(x-1) = 0$

$x = 1, -2$

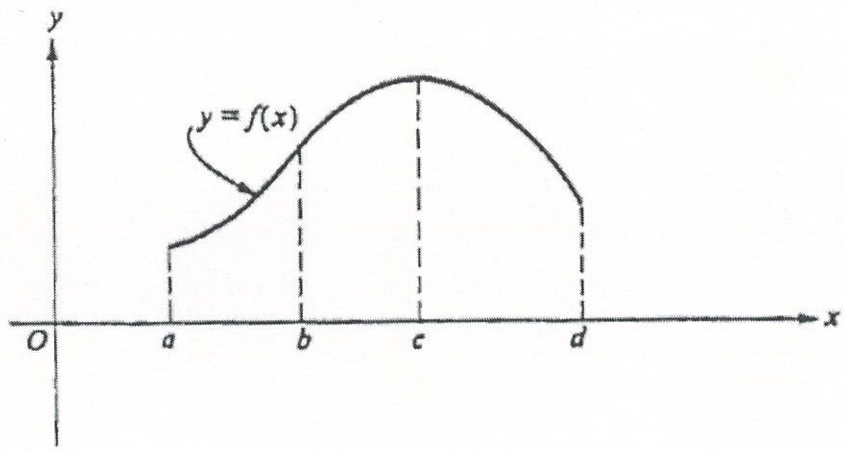
f' + 0 − 0 +

x −2 0 2

a) -3 b) 0 c) 2 d) 4 e) 8

MIN @ $x = 1$

$f(1) = \cancel{4}\ -3$



10. The graph $y = f(x)$ of is shown above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

INC & CONCAVE DOWN $b < x < c$

I. $a < x < b$ II. $b < x < c$ III. $c < x < d$

a) I only b) II only c) III only

d) I and II only e) None of these

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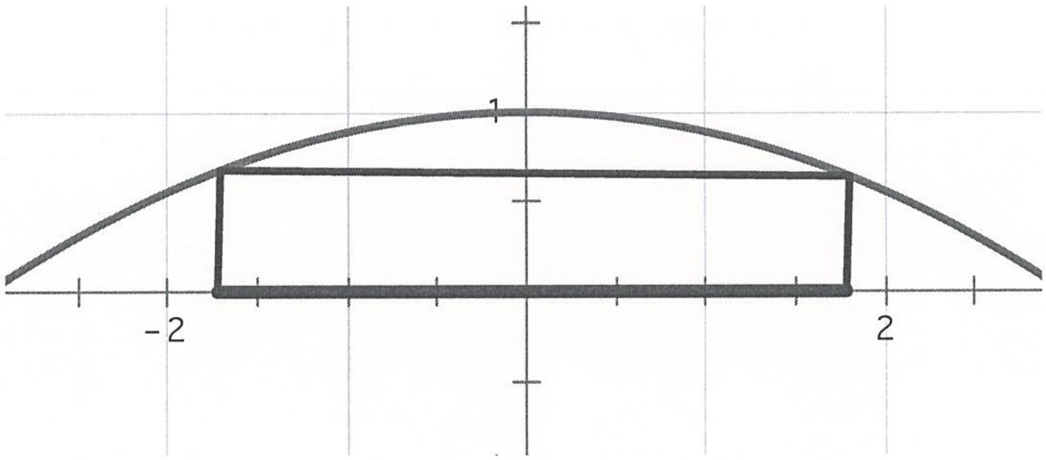
11. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = 1 - \frac{x^2}{9}$, as shown in the figure below. What is the maximum area of the rectangle?



a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{2}{3}$ d) $\frac{4\sqrt{3}}{3}$ e) None of these

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$A = \cancel{2x\sqrt{1-}}\ 2x\left(1 - \frac{x^2}{9}\right) = 2x - \frac{2}{9}x^3$

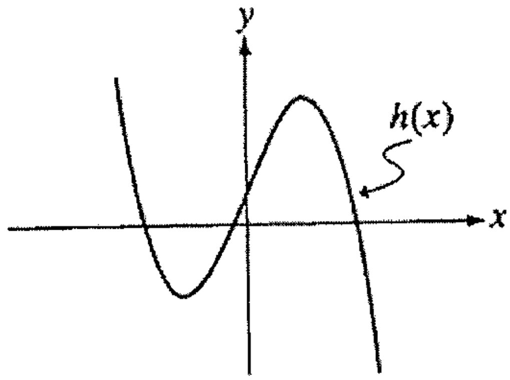
$A' = 2 - \frac{2}{3}x^2 = 0$

$x^2 = 3$

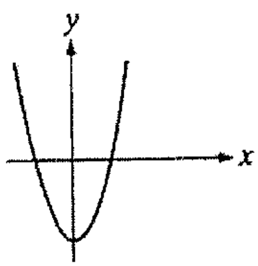
$x = \pm\sqrt{3}$

$A = 2\sqrt{3}\left(1 - \frac{(\sqrt{3})^2}{9}\right) = \frac{4\sqrt{3}}{3}$

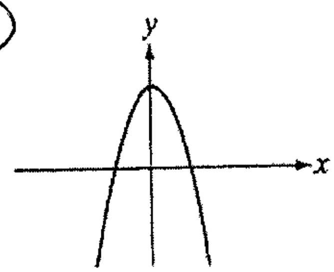
12. Suppose the function of h has the graph shown below. Which of the following could be the graph of $y = h'(x)$?



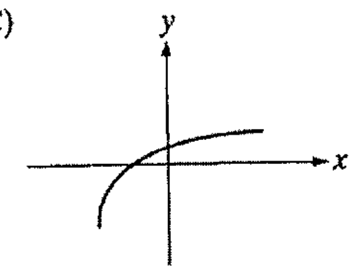
(A)



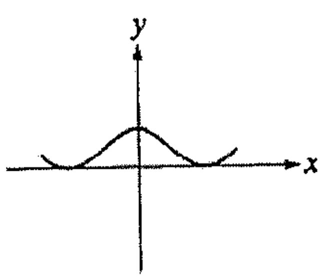
(B)



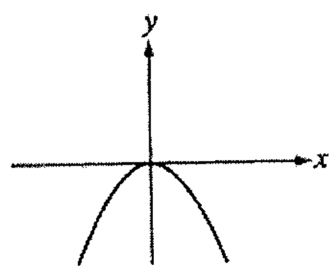
(C)



(D)



(E)



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Score 27

Directions: Show all work.

1. A function f is continuous on the interval $x \in [-3, 3]$ such that $f(-3) = 6$ and $f(3) = 1$. The functions f' and f'' have the properties given below.

x	$-3 \le x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x \le 3$
$f'(x)$	Negative	0	Negative	DNE	Positive
$f''(x)$	Positive	0	Negative	DNE	Negative

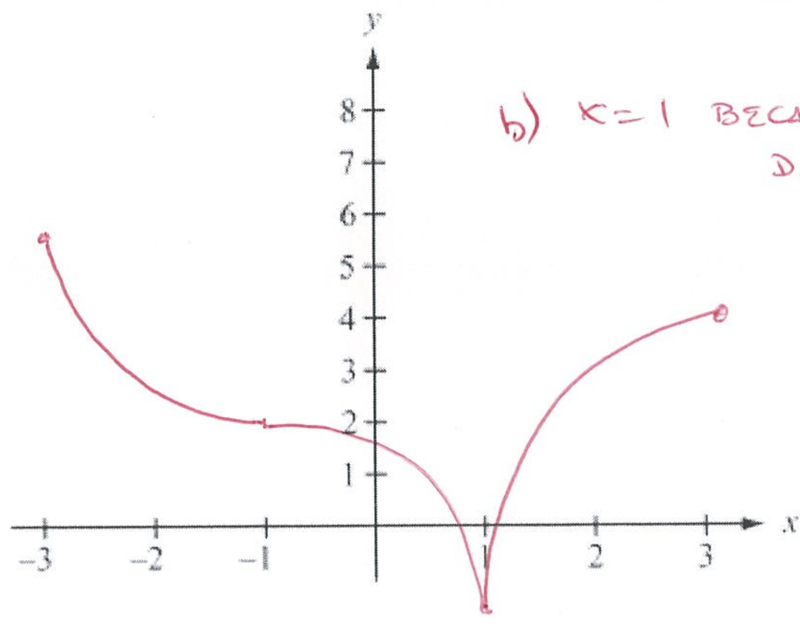
(a) Find all the values of x for which f has a relative maximum or minimum on $x \in [-3, 3]$. Justify your answer.

(b) Find all the values of x for which f has a point of inflection on $x \in [-3, 3]$. Justify your answer.

(e) Sketch a graph of f.

a) Min @ $x = 1$ $f' = 0$ & switches $- +$
Max @ $x = -3, 3$ endpoints

b) $x = 1$ because f' switches decreasing to inc.



2. Consider the velocity equation $v(t)=\dfrac{3t}{t^2-4}$ on $x(2)=4$.

a) For what values is the particle moving right.

V: $-$ DNE $+$ 0 $-$ DNE $+$ on the number line at -2, 0, 2

~~$t \in$ ... 2~~

$t\ [-2, 0] \cup [2, \infty)$

b) What is the acceleration at $t=3$? Show the derivative work.

$$a(t)=\frac{(t^2-4)(3)-3t(2t)}{(t^2-4)^2}$$

$$a(3)=\frac{15-54}{25}=\frac{-39}{25}$$

c) Find the particular position equation.

$$x(t)=\int\frac{3t}{t^2-4}\,dt=\frac{3}{2}\int\frac{du}{u}$$

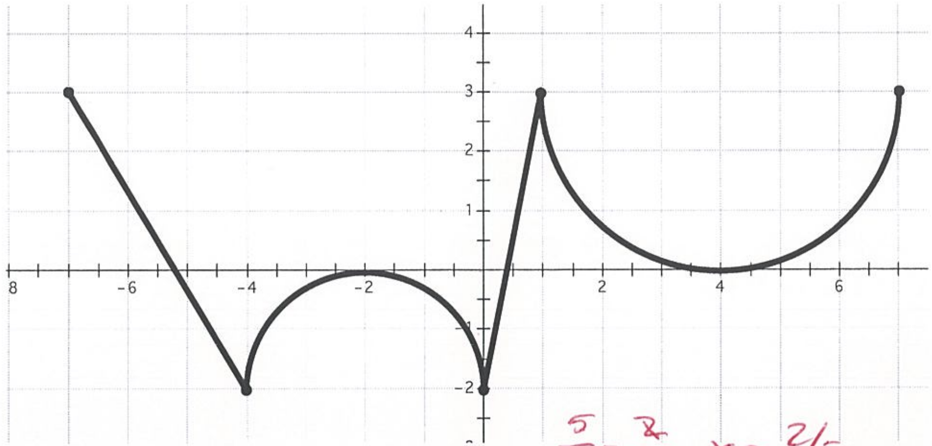
$$=\frac{3}{2}\ln|u|+c$$

$$=\frac{3}{2}\ln|t^2-4|+c$$

$$(2,4)\rightarrow\ 4=\frac{3}{2}\ln 0+c$$

DNE

$\therefore$ THERE IS NO POSITION EQUATION WITH THAT INITIAL VALUE



$\frac{5}{1} = \frac{2}{x}$ $x = 2/5$

3. The graph above is $f'(x)$ on $x \in [-7, 7]$.

a) Identify the x-value(s) of the relative maximums of $y = f(x)$? Justify your answer.

$X = -5.25$ BECAUSE f' SWITCHES + TO −

$X = 7$ BECAUSE f IS INCREASING ($f' > 0$) & STOPS.

b) Identify the x-value(s) of the relative minimums of $y = f(x)$? Justify your answer.

$X = .2/5$ BECAUSE f' SWITCHES FROM − TO +

$X = -7$ BECAUSE f STARTS AND ~~DEC~~ INCREASES ($f' > 0$)

c) Where are the points of inflection on $y = f(x)$? Justify your answer.

$X = -4, -2, 0, 1, 4$

BECAUSE f' SWITCHES FROM INCREASING TO DECREASING OR VICE VERSA.