# AP Calculus BC ’17-18

# Fall Final
# Part IIa

## Calculator Required

Name:

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| Distance $x$ (cm) | 0 | 1 | 5 | 6 | 8 |
|---|---|---|---|---|---|
| Temperature $T(x)$ (°C) | 100 | 93 | 70 | 62 | 55 |

1. A metal wire of 8 cm is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius, of the wire $x$ cm from the heated end. The function $T$ is decreasing and twice differentiable.

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a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate the units.

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b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature using a right-hand Riemann sum with four subintervals indicated by the data on the table. Indicate units of measure.

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c) Find $\int_0^8 T'(x)dx$ and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)dx$ in terms of the temperature of the wire.

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d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every $x$ in the interval $0 < x < 8$ ? Explain your reasoning.

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2. A water tank holds 150 gallons at time $t = 0$. During the time interval $0 \le t \le 12$, water is pumped into the tank at a rate of

$$w(t) = 4e^{\sin\left(\frac{\pi}{12}t\right)} \text{ gallons per hour.}$$

At time $t = 8$, as second pump begins removing water at a rate of

$$R(t) = \frac{13t}{1+2t} \text{ gallons per hour.}$$

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a) How many gallons enter the tank between $t = 0$ and $t = 8$?

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b) At what time between $t = 0$ and $t = 8$ is the amount of water increasing most rapidly?

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c) What is the total amount of water in the tank at $t = 12$ hours?

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d) Is the amount of water increasing or decreasing at time $t = 12$? Justify your answer.

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# End of

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# Part IIb

## No Calculator Allowed

Name:

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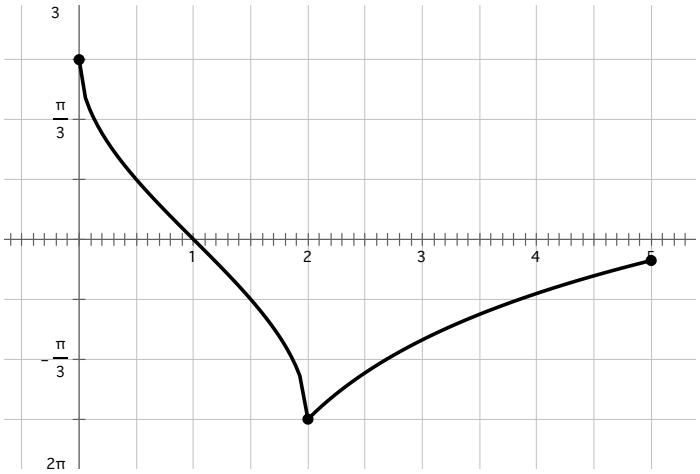
3. The function $f$ is defined as $f(x)=\begin{cases} \sin^{-1}(1-x), \text{ if } 0\le x<2 \\ -\frac{\pi}{2}\quad , \quad if\ x=2 \\ -\frac{\pi}{2}+\ln(x-1), \text{ if } 2<x\le 5 \end{cases}$. The graph is below.

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a) Prove $f(x)$ continuous at $x=2$.

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b) Prove $f(x)$ not differentiable at $x=2$.

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c) Set up, but **do not solve**, an expression for the arc length of $f(x)$ on $x \in [0, 5]$

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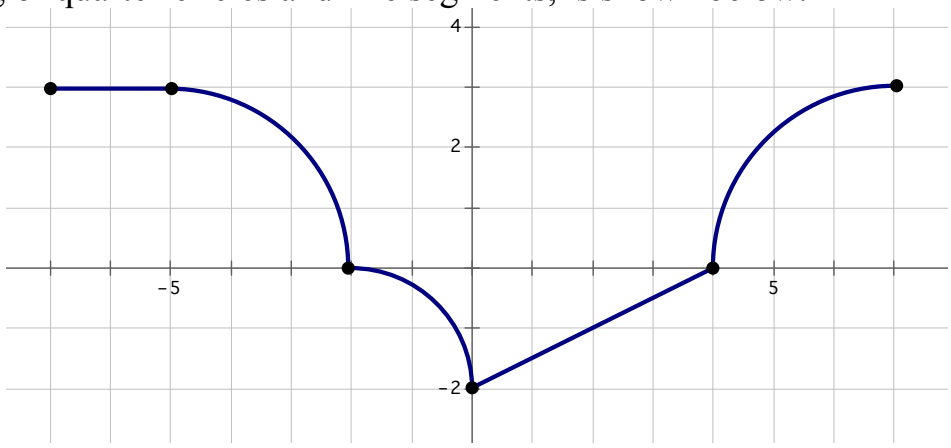
4. Let $g(x)=\int_{-2}^{x} f(t)\,dt$ for $-7\le t\le 7$, where the graph of the function $f$, consisting of quarter-circles and line segments, is shown below.

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a) Find $g(4)$, $g'(4)$, and $g''(4)$.

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b) Find the **average rate of change** of $g(x)$ on $-2\le t\le 4$?

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c) At what $x$-values is $g(x)$ decreasing and concave up? Justify your answer.

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d) Find the $x$-coordinate of the absolute minimum of $g(x)$. Justify your answer.

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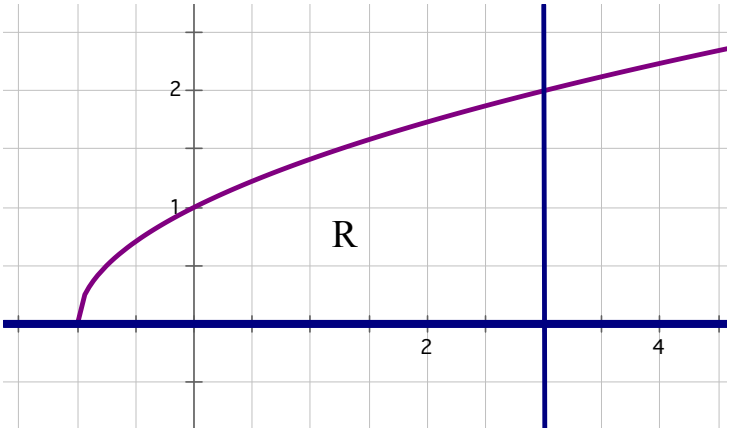
5. Let R be the region bounded by the x-axis, $y=\sqrt{x+1}$, and the line $x=3$ as shown below.

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a) Find the area of region R.

b) Let the base of a solid be the region R. If the cross-sections of the solid perpendicular to the $x$-axis are rectangles that are twice as tall as they are wide, find the volume of the solid.

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c) Set up, but **do not solve**, an expression for the volume of a solid formed if Region R is revolved about the line $x = 4$.

6. Consider the equation $\frac{dy}{dx} = \frac{xy}{x^2+4}$.

Let $y = f(x)$ be the solution to the differential equation above with initial condition $y(0) = 4$.

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a) Find the equation of the line tangent to $y = f(x)$ at $\left(2, 4\sqrt{2}\right)$.

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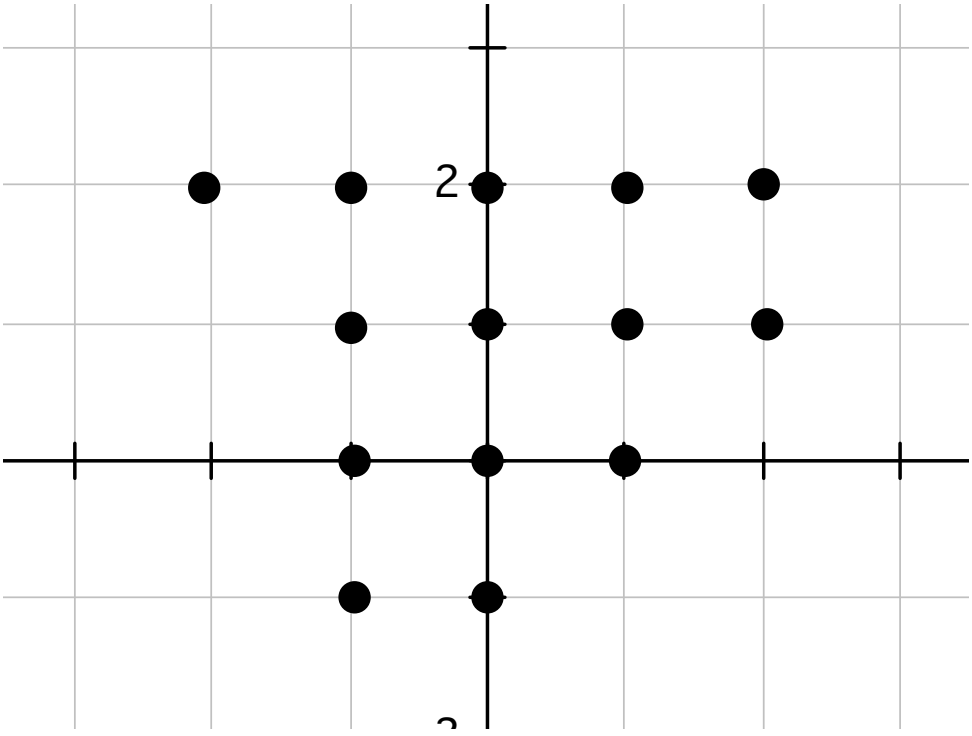
b) On the axes provided, sketch a slope field for the given differential equation at the points indicated.



c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + 4}$ with initial condition $y(0) = 4$.

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