BC Calculus 2017-18
Limit and Continuity Test

Name Solution Key

Score______

1. Let $f(x)=\begin{cases} e^x, \text{ if } x\le 0 \\ \sin x, \text{ if } 0<x \end{cases}$. Which of the following statements is true about f?

$e^0 = 1 \neq 0 = \sin 0$

~~I.~~ f is continuous at $x = 0$.
~~II.~~ f is differentiable at $x = 0$.
III. f has a local maximum at $x = 0$.

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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2. The function f defined on all the Reals such that

$f(x)=\begin{cases} x^2-kx-3 & \text{for } x\le 1 \\ 3x+b & \text{for } x>1 \end{cases}$. For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(a) $k=-1,\ b=2$
(b) $k=1,\ b=3$
(c) $k=1,\ b=4$
(d) $k=1,\ b=-4$
(e) $k=-1,\ b=-4$

$f'(x)=\begin{cases} 2x-k \\ 3 \end{cases}$

$2-k=3 \rightarrow k=-1$

$1+1-3 = 3+b = -1$

$-4 = b$

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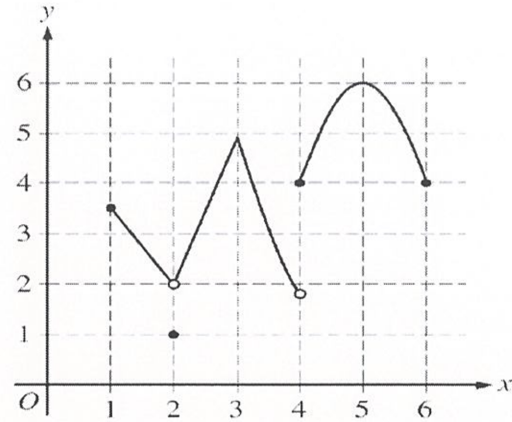
3. The function f is defined on the interval $x \in [-5, 5]$ and has the graph shown below.



Graph of f

Which of the following is (are) true?

I. $\lim_{x \to 2} f(x) = 2$

II. $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = dne$

~~III.~~ $\lim_{x \to 3} f(x) = f(6)$

a) I only b) II only c) III only

d) I and II only e) I and III only

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4. $\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h} =$ $\frac{d}{dx} \cos x \Big|_{x = 3\pi/2} = -\sin \frac{3\pi}{2}$

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) 0 (d) -1 (e) DNE

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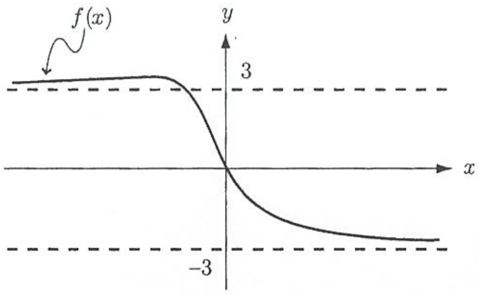
5. The figure below shows the graph of a function $f(x)$ which has horizontal asymptotes of $y = 3$ and $y = -3$. Which of the following statements are false?



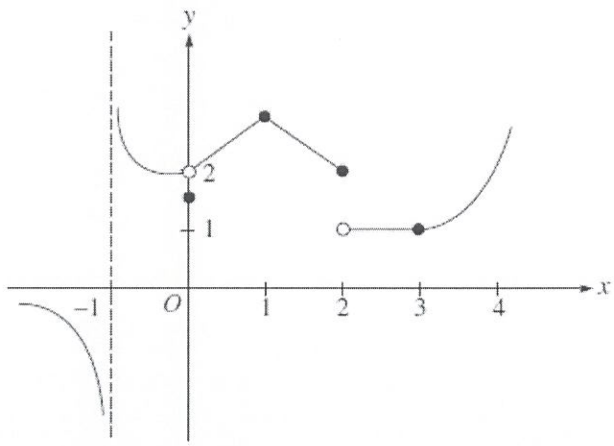
I. $f'(x) < 0$ for all $x \leq 0$ F

II. $\lim_{x \to -\infty} f(x) = 3$ T

III. $\lim_{x \to \infty} f'(x) = -3$ F

(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

6. The function f is shown below. Which of the following statements about he function f, shown below, is false?



a) $\lim_{x\to 0} f(x)$ does not exist

b) $\lim_{x\to 3} f(x)$ exists

c) f is continuous at $x=3$

d) $\lim_{x\to 3}\dfrac{f(x)-5}{x-3}$ does not exists

e) $\lim_{x\to 0} f(x)$ exists

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7. Which of the following improper integrals diverge?

I. $\int_0^{\infty}\frac{1}{1+x^2}\,dx = \tan^{-1}x\Big]_0^{\infty}$ CON

II. $\int_0^{\infty}\frac{1}{x^2}\,dx = -\frac{1}{x}\Big]_0^{\infty}$ DIV

III. $\int_{-1}^{1} u^{-2}\,du$ VA @ 0 $= \frac{u^{-1}}{-1} \Rightarrow$ DIV

(a) II only (b) I and II only (c) I and III only

(d) II and III only (e) I, II, and III

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8. Let $f(x)=\begin{cases} -x+5, & \text{if } x\le -2 \\ x^2+1, & \text{if } -2\le x\le 1 \\ 2x^3, & \text{if } 1<x \end{cases}$. Which of the following statements is true about f?

@ $x=1 \to x^2+1=2=2(1)^3$

T I. f is continuous at $x = 1$. ✓

F ~~II.~~ f is differentiable at $x = 1$. $f' = \begin{cases} -1 \\ 2x \\ 6x^2 \end{cases}$ @ $x=1$ $2 \neq 6$

F III. f has a local maximum at $x = 0$. NO, MIN

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
3	0	0	7	5

9. Given that $f(x)$ is a thrice differentiable, continuous function on the interval $(0, 4)$ with the table values given above. $\lim_{x\to 3}\frac{(x-3)^3}{f(x)}=$ $\overset{L'H}{=} \lim_{x\to 3} \frac{3(x-3)^2}{f'(x)} = \frac{0}{0}$

$\therefore \overset{L'H}{=} \lim_{x\to 3} \frac{6(x-3)^1}{f''(x)} = \frac{0}{7}$

(a) 0 (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$ (e) dne

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10. If a and b are positive constants, then $\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} =$ L'H $\lim_{x\to\infty} \frac{(ax^2+3)b}{(bx+1)2ax} =$

a) 0 b) $\frac{1}{2}$ c) $\frac{ab}{2}$ d) 2 e) ∞

$\lim_{x\to\infty} \frac{ab\cancel{x^2}}{2ab\cancel{x^2}}$

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11. $\lim_{x\to 2} \frac{x^2-4}{\int_2^x \cos(\pi t)\,dt} =$ L'H $\lim_{x\to 2} \frac{2x}{\cos \pi x} = \frac{4}{1}$

a) 0 b) 1 c) 2 d) 4 e) DNE

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12. At $x=3$, the function given by $f(x)=\begin{cases} -x^2, & \text{if } x<-3 \\ 9-6x, & \text{if } -3\le x \end{cases}$ is

9 − 6x is CONT & DIFF @ x=3

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

AP Calculus BC 2017-18
Limit/Continuity Test

Name Solution Key

Score__________.

1. $$f(x)=\begin{cases} 1+\sin x, \text{ if } x<0 \\ 1 \quad , \quad \text{if } x=0 \\ e^{-x}, \text{ if } 0<x \end{cases}$$

a) Is $f(x)$ continuous? Why/Why not?

i) $f(0)$ exists

ii) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (1+\sin x) = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} e^{-x} = 1$

$\lim_{x\to 0} f(x)$ exists

iii) $\lim_{x\to 0} f(x) = f(0)$ $\therefore$ continuous

b) Is $f(x)$ differentiable? Why/Why not?

$$f'(x) = \begin{cases} \cos x & \text{if } x<0 \\ -e^{-x} & \text{if } x>0 \end{cases}$$

$\lim_{x\to 0^-} f(x) = 1 \neq -1 = \lim_{x\to 0^+} f(x)$

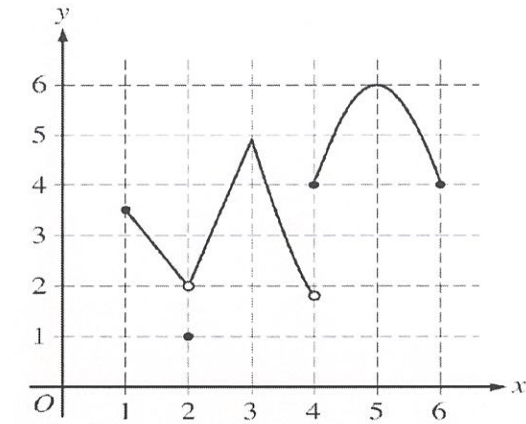
$\therefore$ discontinuous

2. Evaluate $\int_0^\infty \frac{1}{w^2+4}\, dw$

$= \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\frac{w}{2}\Big]_0^\infty$

$= \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\frac{b}{2} - \frac{1}{2}\tan^{-1} 0$

$= \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$



Graph of f

3. For this graph, find

(a) $\lim_{x\to2^-} f(x) = 2$ (b) $\lim_{x\to2^+} f(x) = 2$ (c) $\lim_{x\to3} f(x) = 5$

(d) $\lim_{x\to4^+} f(x) = 4$ (e) $\lim_{x\to5^+} f(x) = 6$ (f) $\lim_{x\to4} f(x) =$ DNE

(g) $f(4) = 4$ (h) $\lim_{x\to1^-} f(x) =$ DNE (i) $f(2) = 1$ (j) $f(3) = 5$

4. $\int_1^2 \frac{x^2dx}{x^3-1} = \frac{1}{3}\int_0^7 \frac{1}{u}du$

$= \frac{1}{3}\lim_{a\to0^+} \ln|u|\Big]_a^7$

$\lim_{a\to0^+} \ln|a|$ DNE $\therefore \int$ IS DIVERGENT.