BC Calculus '17-18 Name Solution Key
Numerical Series Test
No Calculator Allowed Score___________.

1) Which of the following sequences converge?

~~I.~~ $\left\{\dfrac{3n^5}{7n^4-1}\right\}$ $\lim\limits_{n\to\infty} a_n = \infty$ II. $\left\{\dfrac{\sin n}{5^n}\right\}$ DCT $\left(\frac{1}{5}\right)^n$ ~~III.~~ $\left\{\dfrac{n!}{(n+1)!}\right\}$ $\lim\limits_{n\to\infty} a_n = 0$ ~~RATIO TEST~~

(A) I only (B) II only (C) I and III only

(D) II and III only (E) III only

2) Which of the following sequences diverge?

I. $\left\{\dfrac{n}{Lnn}\right\}$ ~~II.~~ $\left\{\left(\dfrac{3e}{13\cos\pi}\right)^n\right\}$ $\left(\frac{7.4}{13}\right)^n < 1$ CON ~~III.~~ $\left\{\dfrac{n!}{(n+2)!}\right\} = \dfrac{1}{(n+2)(n+1)}$

RATIO TEST = 0

(A) I only (B) II only (C) III only

(D) I and II only (E) I, II, and III

3) Which of the following series diverge?

I. $\displaystyle\sum_{n=1}^{\infty}\frac{3^{2n}}{2^{3n}} = \left(\frac{9}{8}\right)^n$ ~~II.~~ $\displaystyle\sum_{n=3}^{\infty}\frac{1}{(n+1)^3}$ CONV DCT $1/n^3$ III. $\displaystyle\sum_{n=2}^{\infty}\frac{n+1}{\sqrt{n^3+2}} \approx \frac{1}{n^{1/2}}$

LCT DIV

(A) I only (B) II only (C) III only

(D) I and III only (E) I, II, and III

4) What are all values of k for which the infinite series $\sum_{n=1}^{\infty}\left(\frac{k}{7}\right)^n$ diverges?

(A) $|k|>7$ (B) $|k|\geq 7$ (C) $|k|>1$

(D) $|k|\geq 1$ (E) $k=0$

$|k|\geq 7$

$\left|\frac{k}{7}\right|\geq 1$

5) If $f(x)=\sum_{n=1}^{\infty}\left(\left(\frac{1}{2}-\tan x\right)^3\right)^n$, then $f\left(\frac{\pi}{4}\right)=$ $\sum_{n=1}^{\infty}\left(\frac{-1}{8}\right)^n \rightarrow S = \frac{-1/8}{1-(-1/8)}$

(A) $\frac{1}{7}$ (B) $\frac{8}{9}$ (C) $-\frac{1}{9}$ (D) 8 (E) divergent $= \frac{-1}{9}$

6) Which of the following series diverge?

I. $\sum_{n=1}^{\infty}\frac{n+2}{n^2+1}$ II. $\sum_{n=1}^{\infty}\frac{n!}{2^n}$ III. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{1+n}\right)^n$ $\lim_{n\to\infty} a_n \neq 0$

LCT to $1/n$

(A) I only (B) III only (C) I and III only

(D) II and III only (E) I, II, and III

7) Which of the following series are convergent?

~~I.~~ $\sum_{n=1}^{\infty} n^{-.9}$ $p<1$ II. $\sum_{n=1}^{\infty}\frac{3^n}{n+5^n}$ DCT $\left(\frac{3}{5}\right)^n$ ~~III.~~ $\sum_{n=1}^{\infty}\frac{n}{1+4n}$ DIV TEST

(A) I only (B) II only (C) III only

(D) I and II only (E) II and III only

8) Which of the following series are conditionally convergent?

I. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}$ ABS II. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$ III. $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$ ABS

(A) I only (B) II only (C) III only

(D) I and II only (E) I and III only

9) Which of the following series are convergent using the Ratio Test?

~~I.~~ $\sum_{n=0}^{\infty} \frac{1}{n^2}$ INCON II. $\sum_{n=0}^{\infty} \frac{n}{3^n}$ ~~III.~~ $\sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n}}$ DIV

(A) I and III only (B) II only (C) III only

(D) I and II only (E) II and III only

10) Which of the following test will establish $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$ is convergent?

I. Direct Comparison Test with $\sum_{n=1}^{\infty} 2n^{-2}$

II. Limit Comparison Test with $\sum_{n=1}^{\infty} n^{-2}$

~~III.~~ Direct Comparison Test with $\sum_{n=1}^{\infty} 2n^{-1}$

(A) I only (B) II only (C) III only (D) I and II only (E) II and III

BC Calculus '17-18
Numerical Series Test
Calculator Allowed

Name SOLUTION KEY

Score__________.

1. Use the Integral Test to determine if $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is convergent or divergent.

$$\int_1^{\infty} x^2 e^{-x^3}\,dx$$

$$= \lim_{b\to\infty}\left(-\frac{1}{3}\right)\int_1^b e^{-x^3}(-3x^2)\,dx$$

$$= \lim_{b\to\infty} -\frac{1}{3}e^{-x^3}\Big]_1^b$$

$$= \lim_{b\to\infty} -\frac{1}{3}e^{-b^3} + \frac{1}{3}e^{-1}$$ = A REAL # $\therefore$ THE $\int$ IS CONVERGENT AND SO IS THE SERIES

2. Use the Ratio Test to determine if $\sum_{n=1}^{\infty} \frac{2^n \cdot n^3}{n!}$ is convergent or divergent.

$$\lim_{n\to\infty}\left|\frac{\frac{2^{n+1}(n+1)^3}{(n+1)!}}{\frac{2^n \cdot n^3}{n!}}\right| = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2\cdot (n+1)^{\cancel{3}\,2}}{\cancel{(n+1)}(\cancel{n!})} \cdot \frac{\cancel{n!}}{\cancel{2^n}n^3}$$

$$= \lim_{n\to\infty} \frac{2(n+1)^2}{n^3} = 0 < 1 \therefore$$

CONVERGENT

3. Determine if $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+4)^2}$ is convergent or divergent.

COMPARE TO $\frac{1}{n^{3/2}}$

$\frac{\sqrt{n}}{(n+4)^2} < \frac{1}{n^{3/2}}$ BECAUSE A LARGE DENOM MAKES A SMALLER FRACTION.

$\frac{1}{n^{3/2}}$ CONVERGES ~~(PST)~~ $(p>1)$ $\therefore$ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+4)^2}$ CONVERGES

BECAUSE IT IS SMALLER THAN A CONVERGENT SERIES

OR

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{(n+4)^2}}{1/n^{3/2}} = \lim_{n\to\infty} \frac{n^{1/2}}{(n+4)^2} \cdot \frac{n^{3/2}}{1} = \lim_{n\to\infty} \frac{n^2}{(n+4)^2} = 1$$

LCT $= 1$ $\therefore$ BOTH SERIES DO THE SAME THING

AND $\frac{1}{n^{3/2}}$ CONVERGES, SO $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+4)^2}$ CONVERGES