BC Calculus '17-18
Parametric/Polar Test

Name Solution Key

1. The velocity of a particle's motion is described by $\langle t^2+t-2,\ 2t^2+3t-2\rangle$. At $t=1$, the particle's position is $(3,-5)$. $y(6)=$

a) 79.167

b) 74.167

c) 185.833

d) 180.833

e) 188.833

$$y(6) = y(1) + \int_1^6 (2t^2+3t-2)\,dt$$
$$= -5 + \left(\tfrac{2}{3}t^3 + \tfrac{3}{2}t^2 - 2t\right]_1^6$$

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2. A curve is described by the parametric equations $x=t^2+2t$ and $y=t^3+t^2$. An equation of the line tangent to the curve at the point determined by $t=1$ is

a) $2x-3y=0$ b) $4x-5y=2$ c) $4x-y=10$

d) $5x-4y=7$ e) $5x-y=13$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2+2t}{2t+2} \qquad m = \left.\frac{dy}{dx}\right|_{t=1} = \frac{5}{4}$$

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3. Find the slope of the line tangent to $r = 2\theta + \cos\theta$ at $\theta = \frac{\pi}{2}$.

a) 0 b) 2 c) 3 d) $\frac{1}{\pi}$ e) $-\frac{1}{\pi}$

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta} = \frac{\pi(0) + 1(1)}{-\pi(1) + 1(0)} = \frac{1}{-\pi}$$

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4. The area of one loop of the graph of the polar equation $r = 2\sin(3\theta)$ is given by which of the following expressions?

a) $4\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\,d\theta$ b) $2\int_0^{\frac{\pi}{3}} \sin(3\theta)\,d\theta$ c) $2\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\,d\theta$

d) $2\int_0^{\frac{2\pi}{3}} \sin^2(3\theta)\,d\theta$ e) $2\int_0^{\frac{2\pi}{3}} \sin(3\theta)\,d\theta$

$$A = \int_0^{\pi/3} \frac{1}{2}(2\sin 3\theta)^2\,d\theta$$

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5. A particle moves on a plane so that its position vector is $p(t)=\left\langle \frac{1}{3}t^3+\frac{1}{2}t^2-2t+7,\ \frac{2}{3}t^3+\frac{3}{2}t^2-2t+\pi^6 \right\rangle$ is at rest when

$v(t)=\langle t^2+t-2,\ 2t^2+3t-2\rangle$

$(t+2)(t-1)=0$ | $(2t+1)(t+2)=0$

$t=1,-2$ | $t=-1/2,-2$

a) $t=1$ only

b) $t=\frac{1}{2}$ only

c) $t=-2$ only

d) $t=1,\ \frac{1}{2}$

e) $t=1,\ \frac{1}{2},\ -2$

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6. An object moves in the xy-plane so that its position at any time t is given by the parametric equations $x(t)=t^4+1$ and $y(t)=\cos\left(\frac{\pi}{2}t\right)$. What is the rate of change of y with respect to x at $(2,0)$?

$t^4+1=2$

$t=\pm 1$

a) $-\frac{\pi}{8}$ b) $-\frac{1}{4}$ c) 4 d) $-\frac{8}{\pi}$

$$\frac{dy}{dx}=\frac{dy/dt}{\frac{dx}{dt}}=\left.\frac{-\frac{\pi}{2}\sin\frac{\pi}{2}t}{4t^3}\right|_{t=\pm 1}=-\pi/8$$

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7. At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle e^{2t}, \sin(3t), \rangle$. What is the acceleration vector of the particle?

~~a)~~ $\langle e^{2t}, \cos(3t) \rangle$

~~b)~~ $\left\langle \frac{1}{2}e^{2t}, \cos(3t) \right\rangle$

c) $\langle 2e^{2t}, 3\cos(3t) \rangle$

d) $\langle 2e^{2t}, -3\cos(3t) \rangle$

$\frac{d}{dt}(e^{2t}) = e^{2t}(2)$

$\frac{d}{dt}(\sin 3t) = (\cos 3t)(3)$

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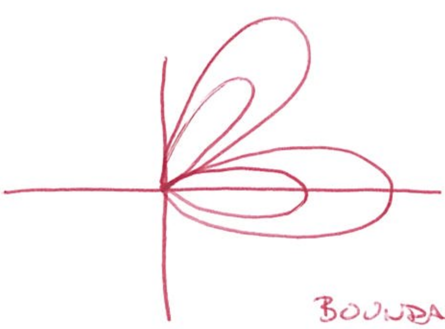
8. What is the total area between the polar curves $r = 4\cos(5\theta)$ and $r = 7\cos(5\theta)$?

a) 14.137 b) 7.069 c) 25.918 d) 51.836



$5\left[\frac{1}{2}\int_{-\pi/10}^{\pi/10} (7\cos 5\theta)^2 - (4\cos 5\theta)^2 \, d\theta\right.$

BOUNDARIES

$4\cos 5\theta = 0$

$\cos 5\theta = 0$

$5\theta = \pm\frac{\pi}{2} \pm 2\pi n$

$\theta = \pm\frac{\pi}{10} \pm \frac{2\pi}{5} n$

9. If $x(t) = 5\sin t$ and $y(t) = 3\cos t$, then $\frac{d^2y}{dx^2} =$

a) $-\frac{3}{5}\cot t$ b) $\frac{3}{5}\tan t$ c) $-\frac{3}{5}\sec^2 t$

d) $\backslash\frac{3}{5}\sec^2 t$ e) $-\frac{3}{25}\sec^3 t$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-3\sin t}{5\cos t} = \frac{-3}{5}\tan t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(-3/5 \tan t\right)}{dx/dt} = \frac{-3/5 \sec^2 t}{5\cos t}$$

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10. A particle's velocity $(x(t), y(t))$ at time $0 \le t \le 10$ is described by the parametric equations $x'(t) = \dfrac{t}{\sqrt{t^2+4}}$ and $y'(t) = \dfrac{5-t}{\sqrt{10t-t^2}}$. At $t=0$, the particle's position is $(2, 0)$

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2 a. At what time is the particle at rest? Justify your answer.

$x'(t) = 0 \rightarrow t = 0$ $\quad y'(t) = 0 \rightarrow t = 5$

NEVER AT REST

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2 b. Find the acceleration at t=5.

$\langle .024, -.2 \rangle$

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1 c. What is the particle's speed at t=5?

$\sqrt{[x'(5)]^2 + [y'(5)]^2} = .928$

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2 d. Is the speed at t=5 increasing or decreasing? Justify your answer.

$$S(t) = \sqrt{\left(\frac{t}{\sqrt{t^2+4}}\right)^2 + \left(\frac{5-t}{\sqrt{10t-t^2}}\right)^2}$$

$S'(5) = +.026$ $\therefore$ INCREASING

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2 e. What is the total distance traveled by the particle on $1 \le t \le 9$?

$$\int_{1}^{9} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt = 8.479$$

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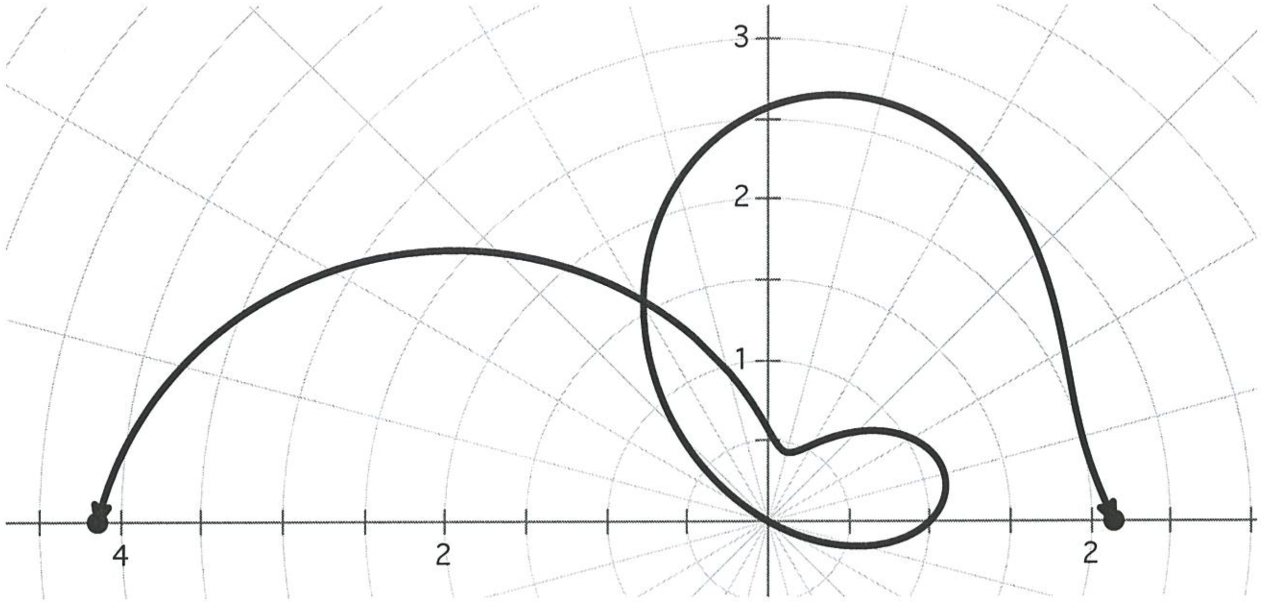
11. The graph below is $r = \theta + \cos 2\theta$ on $\theta \in [-\pi, \pi]$.



a. What are the points in $\theta \in (-\pi, 0)$ (beside the pole) that are furthest and closest to the origin?

4

$$\frac{dr}{d\theta} = 1 - 2\sin 2\theta = 0$$

$$\sin 2\theta = 1/2$$

$$2\theta = \begin{cases} \pi/6 \pm 2\pi n \\ 5\pi/6 \pm 2\pi n \end{cases}$$

$$\theta = \begin{cases} \pi/12 \pm \pi n \\ 5\pi/12 \pm \pi n \end{cases}$$

$$\theta = \frac{-11\pi}{12}, \frac{-7\pi}{12}$$

$r\left(\frac{-11\pi}{12}\right) = -2.580$ FURTHEST

$r\left(\frac{-7\pi}{12}\right) = -2.699$ CLOSEST

b. If the curve crosses itself at $r = -\cancel{1}.547$ and 1.574, find the area of the region enclosed by the loop.

$\theta + \cos 2\theta = -\cancel{1}.547 \rightarrow \theta = \cancel{-1.138}\ -.671$

$\theta + \cos 2\theta = 1.574 \rightarrow \theta = 2.021$

E.C.

$$A = \frac{1}{2}\int_{\cancel{-1.138}\ -.671}^{2.021} (\theta + \cos 2\theta)^2 \, d\theta = .857$$

$= \cancel{1.292}$

c. Find the y-coordinate of the point on $r = \theta + \cos 2\theta$ where $x = -3$. Is r increasing or decreasing at t hose points?

5

$r\cos\theta = -3 = (\theta + \cos 2\theta)\cos\theta$

$\theta = 2.696$

$y = r\sin\theta = (\theta + \cos 2\theta)\sin\theta \Big|_{\theta = 2.696} = 1.433$

$\frac{dr}{d\theta} = 1 - 2\sin 2\theta \Big|_{\theta = 2.696} = -.586 \therefore r$ IS DECREASING