AP Calculus BC '17-18

Spring Final
Part IIa

Calculator Required

Name:

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1. On May 15, the weather in the town of Apcalc changes at a rate of W(t) degrees Fahrenheit per hour. W(t) is a twice-differentiable, increasing and concave up function with selected values in the table below. At midnight, t=0, the weather in Apcalc is 40 degrees Fahrenheit.

t (in hours since midnight)	0	1	3	6	8
W(t) (in degrees Fahrenheit per hour)	-2.3	-2.1	-1.2	1.6	4.5

a) At approximately what rate is the rate of change of the temperature changing at 2am (t=2)? Include units.

b) Use a right Riemann sum with subdivisions indicated by the table to approximate $\int_0^8 W(t)\,dt$. Using correct units, explain the meaning of this value in the context of this problem.

c) Is the approximation from part (c) higher or lower than the actual value of $\int_0^8 W(t)\,dt$? Justify your answer.

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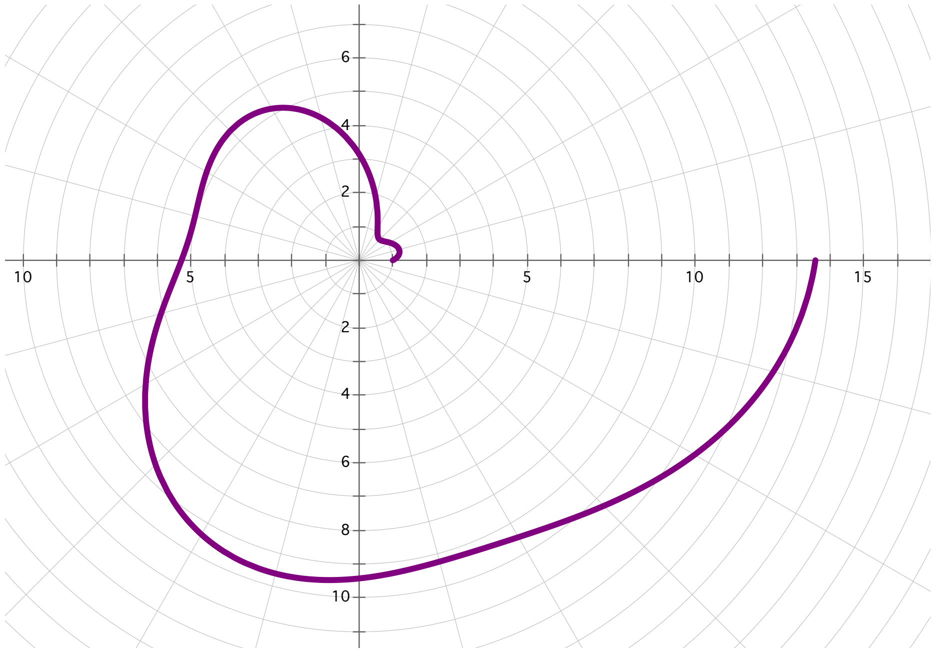
d) In the neighboring town of BC, the weather on the same day is 50 degrees F at midnight. The rate of change of BC's weather is given by $B(t) = 12e^{-t}\sin\left(t^2\right)$ degrees F per hour after midnight. Which town, Apcalc or BC, is warmer at 8am? Justify your answer.



2. The graph of the polar curve $r(\theta) = 2\theta + \cos(3\theta)$ is shown above.

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a) Find the area in the third quadrant enclosed by the coordinate axes and the graph of *r*.

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b) Find the point P on the curve r with x-coordinate 3. Find the angle θ that corresponds to the point P. Show the work that leads to your answers.

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c) A particle travels along r so that its position at time t is $\langle x(t), y(t) \rangle$ and the angle θ increases at π radians per second. Find $\frac{dy}{dt}$ when $\theta = \pi$. Interpret the meaning of your answer in the context of this problem.

End of

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Part IIb

No Calculator Allowed

Name:

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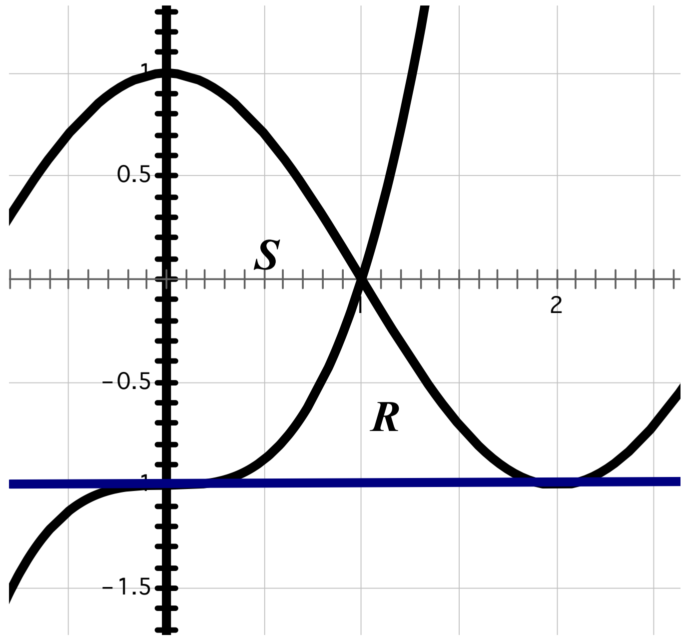

3. Let R and S be the regions shown in the figure above. The regions are bounded by the y-axis and the graphs of $y = x^3 - 1$, $y = \cos\left(\frac{\pi}{2}x\right)$ and the line $y = -1$.

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a) Find the area of S.

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b) Set up, but do not solve, an integral expression to find the volume of the solid formed by rotating R around the y-axis.

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c) Set up, but do not solve, an integral expression to find the volume of the solid with S as its base and cross-sections parallel to the y-axis which are rectangle that are three times as tall as the base is wide.

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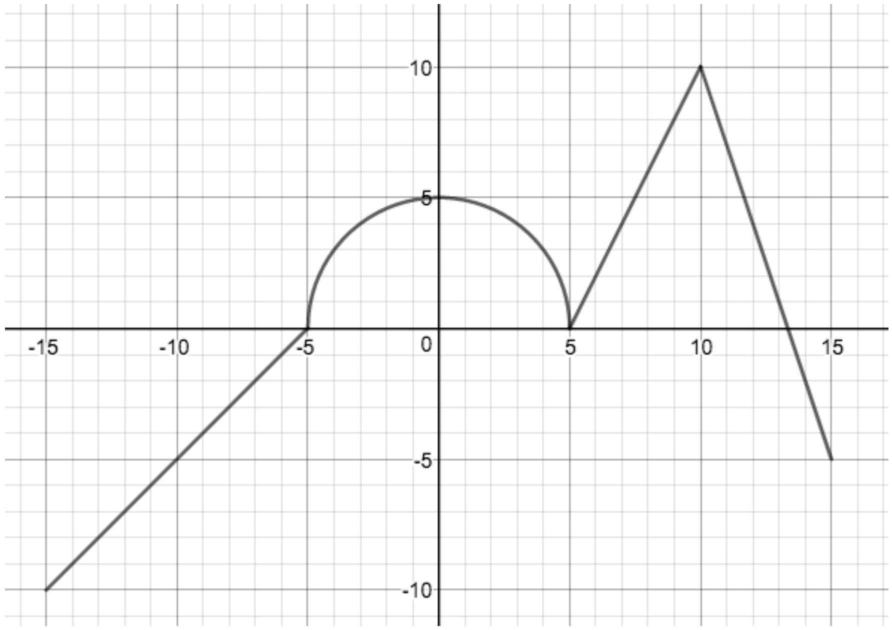
4. The graph of $y = h(x)$ is shown below. Let $g(x) = \int_5^x h(t)\,dt$.



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a) Find $g(10)$, $g'(10)$, and $g''(10)$.

b) On what interval(s), if any, is $g(x)$ both increasing and concave up? Justify your answer.

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c) Write the second degree Taylor polynomial for $g(x)$ at $x = -10$

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5. Consider the differential equation $\frac{dy}{dx} = y\left(x^3 - 4\right)$. Let $y = f(x)$ be the particular solution to this differential equation with initial condition $f(2) = 1$.

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a) Use Euler's Method with two steps of equal size starting at $x = 2$ to approximate $f(4)$. Show the work that leads to your answer.

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b) Find $\frac{d^2y}{dx^2}$ at the point $(2, 1)$.

c) Find the particular solution $y = f(x)$.

6. The function $f(x)$ has a Taylor series about $x = 2$ that converges to $f(x)$ for all *x* in the interval of convergence. It is known that $f(2) = 0$, $f'(2) = 1$, and f has *n*th derivative defined recursively as

$$f^{(n)}(2) = -(n-1) \cdot f^{(n-1)}(2) \text{ for all } n \geq 2.$$

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a) Show that the Taylor series for $f(x)$ is

$$(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3} - \frac{(x-2)^4}{4} + \cdots + \frac{(-1)^{n+1}(x-2)^n}{n} + \cdots$$

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b) Determine if $x = 2$ is at a maximum, a minimum, or neither for $f(x)$. Justify your answer.

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c) Find the radius and interval of convergence for the series for $f(x)$.

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d) Use the Taylor series for $f(x)$ to write the first three terms and the general term of the Taylor series for $g(x) = x \cdot f(x^2 + 2)$.

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