AP Calculus BC ‘17-18 Name________________________________
Taylor Series Test

Directions: Show all work. Score___________.

1. Which of these is the Maclaurin series expansion for the function $h(x)=\frac{\sin(x^3)}{x}$?

a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n-1}}{(2n)!}$ b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+3}}{(2n+1)!}$ c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+2}}{(2n+1)!}$

d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ e) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!}$

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2. If $\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n+1}}{(2n+1)!}$ is the series expansion for $f(x)$, which of these is the series for $f'(x)$?

a) $\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n}}{(2n)!}$ b) $\sum_{n=0}^{\infty}\frac{(-2)^{n-1} x^{2n}}{(2n)!}$ c) $\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty}\frac{(-2)^n (2n+1)x^{2n}}{(2n)!}$ e) $\sum_{n=0}^{\infty}\frac{(-2)^{n-1} x^{2n+2}}{(2n+2)!}$

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3. . Which of these series converge when $x = -1$?

I. $\sum_{n=1}^{\infty} \frac{3x^n}{\sqrt[4]{n}}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ III. $\sum_{n=1}^{\infty} \frac{(1-x)^n}{3^n}$

a) I and II only b) II and III only

c) I and III only d) III only e) I, II, and III

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4. The interval of convergence for the series $\sum_{n=0}^{\infty} \frac{n}{4^n}(x-2)^n$ is

a) $x \in [-4,4)$ b) $x \in (-4,4)$ c) $x \in [-2,6)$

d) $x \in (-2,6)$ e) $x \in (-\infty,\infty)$

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5. $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}} =$

a) $\frac{-2}{e^2 - 2e}$ b) $\frac{-2}{e^2 + 2e}$ c) $\frac{-2}{e+2}$ d) $\frac{e}{e+2}$ e) DNE

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6. $1-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\cdots+\frac{(-1)^n\pi^{2n}}{(2n)!}+\cdots=$

a) 1 b) 0 c) -1.2 d) -1 e) π

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7. Let $P(x)=3-3x^2+6x^4$ be the fourth degree Taylor polynomial for the function f about $x=0$. Then $f^{IV}(0)=$

a) 0 b) $\frac{1}{4}$ c) 6 d) 24 e) 144

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8. The radius of convergence for $\sum_{n=0}^{\infty}\frac{3^n(x-1)^n}{n!}$ is

a) 1 b) 3 c) $\frac{1}{3}$ d) 0 e) ∞

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9. If the third degree Taylor series for $f(x)$ about $x = -3$ is used to approximate $f(-2.7)$, and $f^{III}(x) \leq 2$ and $f^{IV}(x) \leq 5$ for all x-values, what is the Lagrange Error bound?

a) 0.001

b) 0.009

c) 0.062

d) 0.100

e) 0.208

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10. The function g has derivatives of all orders, and the Maclaurin series for g is $\sum_{n=0}^{\infty} \frac{x^n}{3n+1} = 1 + \frac{x}{4} + \frac{x^2}{7} + \cdots + \frac{x^{n+1}}{3n+1} + \cdots$

a) What is $g'''(0)$?

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b) Write the first three nonzero terms and the general term for the Maclaurin series for $g'(3x)$

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c) Write the first 3 nonzero terms of the series for $\sin x$ and use them to find the 4^{th} degree Maclaurin polynomial for the product $\sin x \cdot g(x)$

11. A function f has derivatives of all orders at $x=1$ given by

$$f^{(n)}(1)=\frac{(-1)^{n+1}n!}{2^n}.$$

a) Write the first 4 nonzero terms and the general term for the Taylor series for f centered at $x=1$ (Recall, $0!=1$)

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b) Find the radius and interval of convergence for the series you found in part a).

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c) Use the first four nonzero terms of the Taylor series to approximate $f(1.2)$, and show that this approximation is within 0.0001 of the actual value of $f(1.2)$.

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d) Rewrite $f(x)$ as a geometric series and find the actual value of $f(1.2)$.
