BC Calculus '17-18
Volume Test

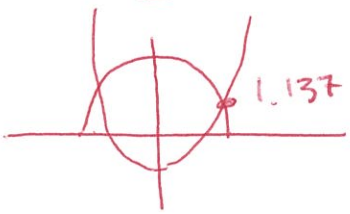
Name Solution Key

Score___________

1. The area of the region enclosed by $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$ is given by

(a) 2.525 (b) 4.049 (c) 4.328

(d) 5.050 (e) 6.289



$$A = \int_{-1.137}^{1.137} \left[\sqrt{4-x^2} - (e^{x^2} - 2)\right] dx$$

---

2. The length of a curve $y = f(x)$ between $x=a$ to $x=b$ is given by $\int_a^b \sqrt{e^{2x} + 2e^x + 2}\, dx$. Therefore, $f(x) =$

(a) $2e^{2x} + 2e^x$ (b) $\frac{1}{2}e^{2x} + 2e^x + 2x$

(c) $e^x - x + 3$ (d) $e^x + 1$

(e) $e^x + x - 2$

$$1 + \left(\frac{dy}{dx}\right)^2 = e^{2x} + 2e^x + 2$$

$$\left(\frac{dy}{dx}\right)^2 = e^{2x} + 2e^x + 1 = (e^x + 1)^2$$

$$\frac{dy}{dx} = e^x \pm 1$$

$$\int dy = \int (e^x + 1)\, dx$$

$$y = e^x + x + c$$

3. Let R be the region in the first quadrant bounded by $y=(x-3)^2$, $y=0$ and $x=0$. What is the volume of the solid generated when R is rotated about the x-axis?

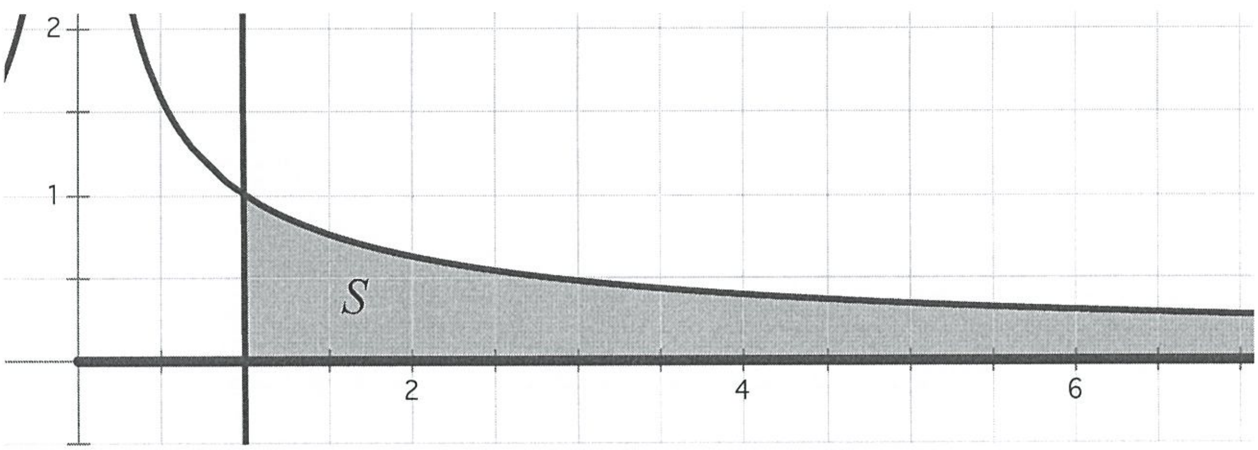
$V = \pi \int_a^b (R)^2 dy$

a) $\pi\int_0^3 (x-3)^2\,dx$ b) $\pi\int_0^3 (x-3)^4\,dx$

c) $2\pi\int_0^3 (x-3)^2\,dx$ d) $2\pi\int_0^3 x(x-3)^2\,dx$

$y = 0 \;\&\; x = 3$

e) $2\pi\int_0^3 x(x-3)^4\,dx$



4. The region S (shown above) is bounded by $y=\dfrac{1}{\sqrt[3]{x^2}}$, the x-axis, and the line $x=1$. There is not upper bound (i.e., $x\to\infty$)A solid is formed by revolving region S about the x-axis. The volume of the solid is

a) 0 b) 1 c) π d) undefined

e) none of the above

$V = \pi\int_1^\infty \frac{1}{x^{4/3}}\,dx = -3\pi x^{-1/3}\Big]_1^\infty$

$0 - (-3\pi) = 3\pi$

$y = \sin x$

5. Let R be the region in the first quadrant bounded by $x = \sin^{-1} y$, the x-axis, and $x = \frac{\pi}{2}$. Which of the following integrals gives the volume of the solid generated when R is rotated about the x-axis?

(a) $\pi \int_0^{\pi/2} y^2 \, dy$

(b) $\pi \int_0^1 \left(\sin^{-1} y\right)^2 dy$

(c) $\pi \int_0^{\pi/2} \left(\sin^{-1} y\right)^2 dy$

(d) $\pi \int_0^1 (\sin x)^2 \, dx$

(e) $\pi \int_0^{\pi/2} (\sin x)^2 \, dx$

DISK

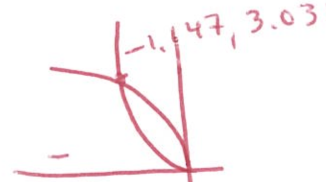
---

6. The base of a solid is the region enclosed by $y = (x-3)^2$ in Quadrant I. If each cross-section of the solid perpendicular to the x-axis is a square, the volume of the solid is

a) 9 b) 9π c) 27.3 d) 48.6 e) 48.6π

$$V = \int_0^3 (x-3)^4 \, dx = \frac{1}{5}(x-3)^5 \Big]_0^3$$

$$= 0 - \left(\frac{-243}{5}\right) = 48.6$$



7. Let T be the region bounded by $y=-2x^3$ and $y=\sqrt{-8x}$.

a) Find the volume of the solid generated when T is rotated about the x – axis. Show the anti-differentiation steps.

$$V = \pi \int_{-1.149}^{0} \left[\left(\sqrt{-8x}\right)^2 - \left(-2x^3\right)^2\right] dx$$

$$= \pi \int_{-1.149}^{0} \left(-8x - 4x^6\right) dx$$

$$= \pi \left[-4x^2 - \frac{4}{7}x^7\right]_{-1.149}^{0}$$

$$= 11.844$$

---

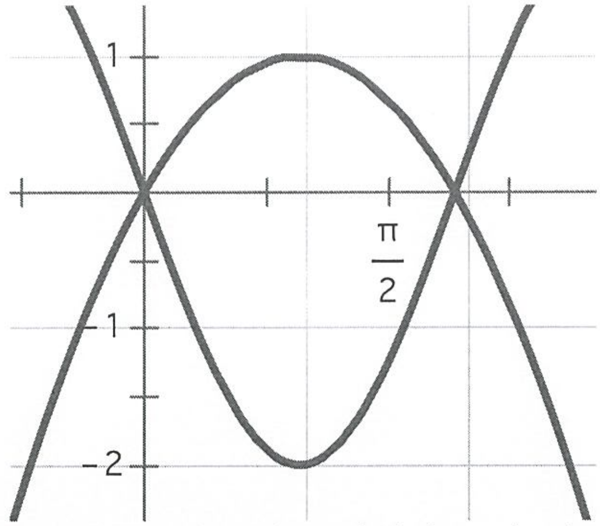
b) Find the volume of the solid generated when T is rotated about the y – axis. Show the anti-differentiation steps.

$$y = -2x^3 \rightarrow x = \left(-\frac{1}{2}y\right)^{1/3}$$

$$y = \sqrt{-8x} \rightarrow x = -\frac{1}{8}y^2$$

$$V = \pi \int_{0}^{3.031} \left[\left(-\frac{1}{2}y\right)^{2/3} - \left(-\frac{1}{8}y^2\right)^2\right] dy$$

$$= \pi \left[\frac{3}{5}\left(\frac{1}{2}\right)^{2/3} y^{5/3} - \frac{1}{320}y^5\right]_{0}^{3.031} = 5.027$$



8. Let S be the region shown above bounded above by the graph of $y = -2\sin\left(\frac{\pi}{2}x\right)$ and below the graph of $y = 2x - x^2$.

a) Find the volume of the solid generated when S is revolved about the line $y = 2$.

$$V = \pi \int_0^2 (2-2x+x^2)^2 - \left(2+2\sin\tfrac{\pi}{2}x\right)^2 dx$$

$$= 57.970$$

---

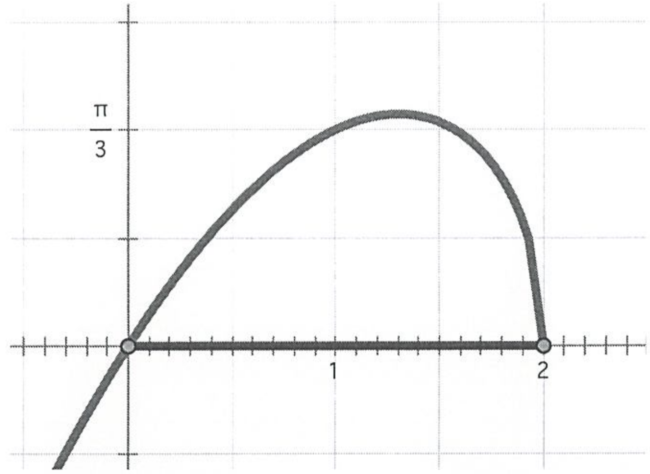
b) Let the base of the solid be the region S. Find the volume of the solid where the cross-sections perpendicular to the x-axis are rectangles that are three times as tall as they are wide.

$$V = 3\int_0^2 \left[\left(2\sin\tfrac{\pi}{2}x\right) - (2x-x^2)\right]^2 dx$$

$$= \cancel{51.446} \quad = 27.585$$

---

9. Let Q be the region bounded by $y = x\cos^{-1}\left(\frac{x}{2}\right)$, and ~~$x$~~ $y = 0$.



a) Find the volume of the solid generated when R is rotated about the line $y = -2$.

$$V = \pi \int_0^2 \left(x\cos^{-1}\left(\tfrac{x}{2}\right) + 2\right)^2 - (+2)^2 \, dx$$

$$= \cancel{74.519} \; 24.253$$

b) Find $\frac{dy}{dx}$.

$$= x \frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{x}{2}\right)$$

$$= \frac{-x}{\sqrt{4-x^2}} + \cos^{-1}\left(\frac{x}{2}\right)$$

c) Find the perimeter of region Q.

$$P = 2 + \int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}} + \cos^{-1}\frac{x}{2}\right)^2}$$

$$= \cancel{5.340}$$

$$= 2 + \lim_{b \to 2^+} \int_0^b \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}} + \cos^{-1}\frac{x}{2}\right)^2}$$

$$= 5.240$$