BC Calculus '18-19
Integration Techniques Test

name Solution Key

Score___________

1. $\int \frac{3x-5}{x^2-7x-18}\,dx =$ $\int \frac{A}{x-9} + \frac{B}{x+2}\,dx$

$A(x+2)+B(x-9)=3x-5$
$x=-2 \rightarrow \quad B=1$
$x=9 \rightarrow \quad A=2$

a) $\ln\left|\frac{x+2}{(x-9)^2}\right|+c$

b) $\ln\left|\frac{(x-9)^2}{x+2}\right|+C$

c) $\ln\left|(x-9)(x+2)^2\right|+C$

[✓ d)] $\ln\left|(x-9)^2(x+2)\right|+C$

e) $\frac{1}{2}\ln\left|\frac{x-9}{x+2}\right|+C$

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2. $\int (x^3)\sqrt{1-x^2}\,dx$

$u=1-x^2$ ~~$u=x^2$~~ $x^2=u+1$
$du=-2x\,dx$

$-\frac{1}{2}\int \cancel{x}\,x^2\sqrt{1-x^2}\,(2x\,dx)$

$=-\frac{1}{2}\int (u+1)u^{1/2}$

$=-\frac{1}{2}\left[\frac{u^{5/2}}{5/2}+\frac{u^{3/2}}{3/2}\right]+c$

a) $\frac{x^4}{2}\cdot\frac{(1-x^2)^{3/2}}{3}+c$

b) $\frac{1}{2}(1-x^2)^{1/2}+\frac{1}{3}(1-x^2)^{3/2}+c$

[✓ c)] $-\frac{1}{3}(1-x^2)^{3/2}-\frac{1}{5}(1-x^2)^{5/2}+c$

d) $\frac{1}{3}(1-x^2)^{3/2}+\frac{1}{5}(1-x^2)^{5/2}+c$

e) $\frac{1}{3}(1-x^2)^{3/2}+c$

3. For $\int \csc^3 x \cot^5 x\, dx$, the correct u-substitution is

[✓ a)] $u = \csc x$

b) $u = \cot x$

c) either $u = \csc x$ or $u = \cot x$

d) neither $u = \csc x$ nor $u = \cot x$

e) convert to sine and cosine

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4. $\int x^3 \ln x\, dx =$ $\quad u = \ln x \quad dv = x^3\, dx$

$du = 1/x\, dx \quad v = \frac{1}{4}x^4$

a) $\frac{x^4}{4}(4\ln x - 1) + c$ [✓ b)] $\frac{x^4}{16}(4\ln x - 1) + c$ c) $\frac{x^2}{4}(\ln x - 1) + c$

d) $3x^2\left(\ln x - \frac{1}{2}\right) + c$ e) $x^2(3\ln x + 1) + c$

$$\int = \frac{x^4}{4}\ln x - \int \frac{1}{4}x^3\, dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + c$$

5. What is the best method to evaluate $\int \frac{dx}{x\left(4x^2-9\right)}$?

a) Integration by Parts b) Substitution [✓ c)] Partial Fractions

d) Completing the Square e) Formula

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6. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{4000}P\left(400-P\right)$, where $P(0) = 100$. What is the end behavior of $P\left(t\right)$?

$\lim_{t\to\infty} P(t) = A = 400$

a) 10

b) 100

c) 200

[✓ d)] 400

e) 4000

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7. Which of the following statements are true?

✗ I. $\int \frac{1}{x\sqrt{16-x^2}}\,dx = \frac{1}{4}\sec^{-1}\frac{x}{4}+c$

[✓ II.] $\int \cot x\,dx = \ln|\sin x| + c$

✗ III. $\int \left(\frac{e^x}{\tan e^x}\right) dx = \ln|\sec e^x| + c$

a) I only
[✓ b)] II only
c) III only
d) I and II only
e) I and III only
f) I, II, and III

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8. $\int x^2 \sin x\ dx =$

a) $-x^2\cos x - 2x\sin x - 2\cos x + c$
b) $-x^2\cos x + 2x\sin x - 2\cos x + c$
[✓ c)] $-x^2\cos x + 2x\sin x + 2\cos x + c$
d) $-\frac{x^3}{3}\cos x + c$
e) $2x\cos x + c$

$+$	x^2	$(-\cos x)$
$-$	$2x$	$(-\sin x)$
$+$	2	$(+\cos x)$

9. $\int \frac{x^2-4}{x^2+4}\,dx =$ $\int 1 - \frac{8}{x^2+4}\,dx$

$$\begin{array}{r} 1 \\ x^2+4 \overline{) x^2-4} \\ -(x^2+4) \\ \hline -8 \end{array}$$

a) $\frac{1}{2}\tan^{-1}\frac{x}{2}+c$

b) $\ln\left|x^2+4\right|+c$

c) $\ln\left|x^2+4\right|+\frac{1}{2}\tan^{-1}\frac{x}{2}+c$

[✓ d)] $x-4\tan^{-1}\frac{x}{2}+c$

e) $x-8\tan^{-1}\frac{x}{2}+c$

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1. $$\int \frac{x^3+x^2}{x^3-5x^2-4x+20}\,dx$$

$$= \int 1 + \frac{6x^2+4x+20}{x^2(x-5)-4(x-5)}\,dx$$

$$= \int\left(1 + \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x-5}\right)dx$$

$$x^3-5x^2-4x+20 \overline{\big)\, x^3+x^2}$$ with quotient 1

$$x^3-5x^2-4x+20$$

$$A(x+2)(x-5) + B(x-2)(x-5) + C(x+2)(x-2) = 6x^2+4x+2$$

$x=2 \Rightarrow A=-1$

$x=-2 \Rightarrow B=-1/7$

$x=5 \Rightarrow C=50/7$

$$= \int 1\,dx - \int \frac{1}{x-2}\,dx - \frac{1}{7}\int \frac{1}{x+2}\,dx + \frac{50}{7}\int \frac{1}{x-5}\,dx$$

$$= x - \ln|x-2| - \frac{1}{7}\ln|x+2| + \frac{50}{7}\ln|x-5| + C$$

⊥ TO THE X-AXIS

2. Find the volume of a solid formed by cross-sections $\wedge$ where the base is the region bounded by $y=0$, $x=0$, $x=\frac{\pi}{2}$, and $y=2x\sqrt{\cos x}$, and where the cross-sections are rectangles where the height is half the base edge. Show the anti-differentiation.

$$V=\int_0^{\pi/2}\frac{1}{2}\left(2x\sqrt{\cos x}\right)^2dx$$

$$=\int_0^{\pi/2}2x^2\cos x\,dx$$

u	V
$2x^2$	$\sin x$
$4x$	$-\cos x$
4	$-\sin x$

$$=2x^2\sin x+4x\cos x-4\sin x\Big]_0^{\pi/2}$$

$$=\left[\frac{\pi^2}{2}+0-4\right]-\left[0+0-0\right]$$

$$=\frac{\pi^2}{2}-4=$$

3. $\int_{-\infty}^{\overset{-3}{0}} \frac{dx}{x^2+6x+13}$

$$= \lim_{a \to -\infty} \int_a^{-3} \frac{dx}{x^2+6x+9+4}$$

$$= \lim_{a \to -\infty} \int_a^{0} \frac{dx}{(x+3)^2+4}$$

$$= \lim_{a \to -\infty} \frac{1}{2} \mathrm{TAN}^{-1} \frac{x+3}{2} \Big]_a^{-3}$$

$$= \frac{1}{2} \mathrm{TAN}^{-1} 0 - \frac{1}{2} \lim_{a \to -\infty} \frac{a+3}{2}$$

$$= 0 - \frac{1}{2} \left(\frac{-\pi}{2} \right)$$

$$= \frac{\pi}{4}$$