AP Calculus BC '18-19
Anti-Derivative Test

Name Solution Key

Score 125

1. Which of the following statements are true?

~~I.~~ $\displaystyle\int \frac{1}{x\sqrt{16-x^2}}\,dx = \frac{1}{4}\sec^{-1}\frac{x}{4}+c$

II. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$

~~III.~~ $\displaystyle\int \left(\frac{e^x}{\tan e^x}\right) dx = \ln|\sec e^x| + c$

a) I only b) II only c) III only

d) I and II only e) I and III only f) All of these

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2. $\displaystyle\int (x-1)\sqrt{x}\,dx =$ $\displaystyle\int \left(x^{3/2} - x^{1/2}\right) dx = \frac{x^{5/2}}{5/2} - \frac{x^{3/2}}{3/2}$

a) $\displaystyle\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + c$ b) $\displaystyle\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + c$ c) $\displaystyle\frac{1}{2}x^2 - x + c$

d) $\displaystyle\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + c$ e) $\displaystyle\frac{1}{2}x^2 + 2x^{3/2} + c$

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3. For $\int \csc^3 x \cot^5 x \, dx$, the correct u-substitution is

a) $u = \csc x$

b) $u = \cot x$

c) either $u = \csc x$ or $u = \cot x$

d) neither $u = \csc x$ nor $u = \cot x$

e) to convert to sine and cosine

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4. $\int (x^3)\sqrt{1+x^2}\, dx$

$u = 1+x^2$ $\quad x^2 = u-1$

$du = 2x\,dx$

a) $\frac{x^4}{2} \cdot \frac{(1+x^2)^{3/2}}{3} + c$

b) $\frac{1}{2}(1+x^2)^{1/2} + \frac{1}{3}(1+x^2)^{3/2} + c$

c) $-\frac{1}{3}(1+x^2)^{3/2} + \frac{1}{5}(1+x^2)^{5/2} + c$

d) $\frac{1}{3}(1+x^2)^{3/2} - \frac{1}{5}(1+x^2)^{5/2} + c$

e.) $\frac{1}{3}(1+x^2)^{3/2} + c$

$\frac{1}{2}\int x^2 (1+x^2)^{1/2} \, 2x\,dx$

$= \frac{1}{2}\int (u-1)\, u^{1/2}\, du$

$= \frac{1}{2}\int (u^{3/2} - u^{1/2})\, du$

$= \frac{1}{2}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right] + C$

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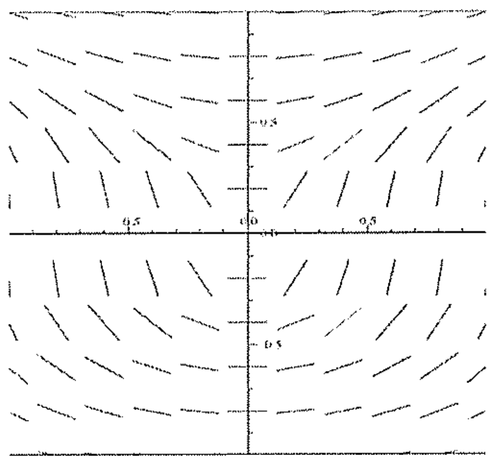
5. Which of the following equations might be the solution to the slope field shown in the figure below?



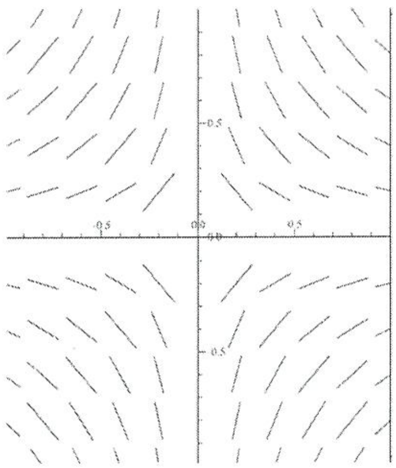
a) $y = 4x - x^3$ b) $y = -\cos x$ c) $y = \sec x$

d) $x = -y^2$ e) $x = -y^3$

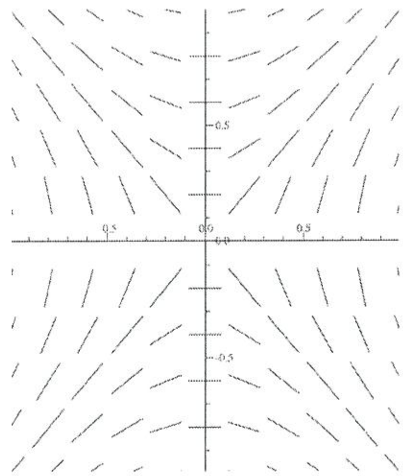
6. Which of the slope field shown below corresponds to $\frac{dy}{dx} = yx$?

$x = 0 \rightarrow m = 0$

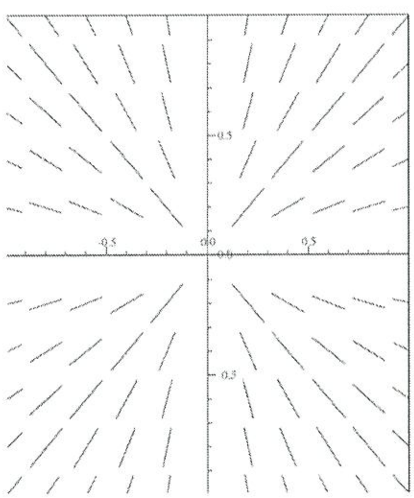
$y = 0 \rightarrow m = 0$



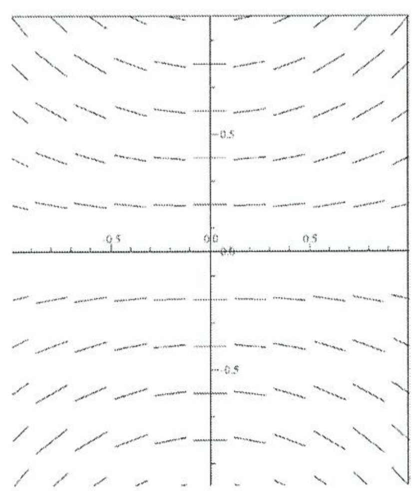
a)



b)



c)



d)

7. Identify is the first mistake (if any) in this process:

$$\frac{dy}{dx} = xy + x$$

Step 1: $\frac{1}{y+1} dy = x dx$

Step 2: $\ln|y+1| = \frac{1}{2}x^2 + c$

Step 3: $|y+1| = e^{x^2} + c$

Step 4: $y = e^{x^2} + c$

a) Step 1 b) Step 2 c) Step 3

d) Step 4 e) There is no mistake.

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8. $\int \left(\sin^5 t \cos^7 t\right) dt$

$u = \sin t$ $\cos^2 t = 1 - \sin^2 t$

$du = \cos t \, dt$ $= 1 - u^2$

$= \int u^5 (1-u^2)^3 \, du$

$= \int u^5 (1 - 3u^2 + 3u^4 - u^6) \, du$

$= \int (u^5 - 3u^7 + 3u^9 - u^{4}) \, du$

$= \frac{1}{6}u^6 - \frac{3}{8}u^8 + \frac{3}{10}u^{10} - \frac{1}{12}u^{12} + C$

$= \frac{1}{6}\sin^6 t - \frac{3}{8}\sin^8 t + \frac{3}{10}\sin^{10} t - \frac{1}{12}\sin^{12} t + C$

9. $\int \frac{1}{(1-x)^2} + \frac{1}{1+x^2} + \frac{x}{1+x^2}\,dx$

$= -\int \frac{\cancel{dx}}{u^2}\,du + \int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$= -\frac{u^{-1}}{-1} + \text{TAN}^{-1}x + \frac{1}{2}\ln(x^2+1) + c$

$= \frac{1}{1-x} + \text{TAN}^{-1}x + \frac{1}{2}\ln|x^2+1| + c$

10. $\frac{1}{3}\int \frac{3y^5\,dy}{\sqrt{y^3+5}}$ $\quad u = y^3+5 \quad du = 3y^2\,dy \quad y^3 = u-5$

$= \frac{1}{3}\int (u-5)u^{-1/2}\,du$

$= \frac{1}{3}\int (u^{1/2} - 5u^{-1/2})\,du$

$= \frac{1}{3}\left[\frac{u^{3/2}}{3/2} - \frac{5u^{1/2}}{1/2}\right] + c$

$= \frac{2}{9}(y^3+5)^{3/2} - \frac{10}{3}(y^3+5)^{1/2} + c$

11. The acceleration of a particle is described by $a(t) = 36t^2 - 12t + 8$. Find the distance equation for $x(t)$ if $v(1) = 1$ and $x(1) = 3$.

$$v = \int (36t^2 - 12t + 8)\, dt$$

$$= \cancel{18}\ 12t^3 - 6t^2 + 8t + C_1$$

$$(1,1) \Rightarrow 1 = 12 - 6 + 8 + C_1 \Rightarrow C_1 = -13$$

$$x(t) = \int (12t^3 - 6t^2 + 8t - 13)\, dt$$

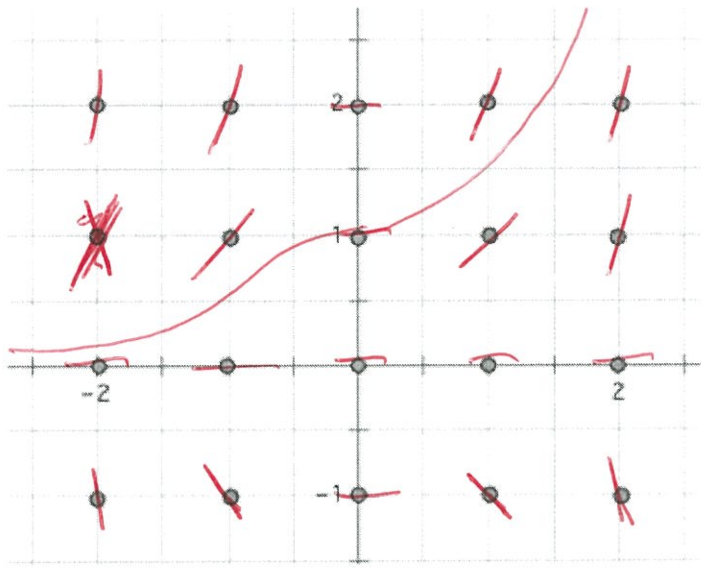
$$= 3t^4 - \cancel{2}\,2t^3 + 4t^2 - 13t + C_2$$

$$(1,3) \Rightarrow 3 = 3 - 2 + 4 - 13 + C_2 \Rightarrow C_2 = 11$$

$$x(t) = 3t^4 - 2t^3 + 4t^2 - 13t + 11$$

12. Given the differential equation, $\frac{dy}{dx} = x^2 y$

a. On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



b. If the solution curve passes through the point (0, 1), sketch the solution curve on the same set of axes as your slope field.

13. Find the particular solution $w = f(t)$ that passes through $\left(0, -\frac{\pi}{4}\right)$ if

$$\frac{dw}{dt} = t^2 \tan(3w)$$

$$\frac{1}{\tan 3w}\, dw = t^2\, dt$$

$u = 3w$

$du = 3dw$

$$\frac{1}{3}\int \cot 3w \; 3dw = \int t^2\, dt$$

$$\frac{1}{3} \ln|\sin 3w| = \frac{t^3}{3} + C$$

$$\ln|\sin 3w| = t^3 + C$$

$$|\sin 3w| = e^{t^3 + C}$$

$$\sin 3w = Ke^{t^3}$$

$$(0, -\pi/4) \Rightarrow \sin \frac{-3\pi}{4} = Ke^0$$

$$-\frac{1}{\sqrt{2}} = K$$

$$\sin 3w = -\frac{1}{\sqrt{2}} e^{t^3}$$

$$3w = \sin^{-1}\left(-\frac{1}{\sqrt{2}} e^{t^3}\right)$$

$$w = \frac{1}{3} \sin^{-1}\left(-\frac{1}{\sqrt{2}} e^{t^3}\right)$$