AP Calculus BC ’18-19
Integral Test

Name Solution Key

Score____________

NO CALCULATOR ALLOWED

$u = 5 - x \quad u(1) = 4$
$du = -dx \quad u(4) = 1$

If $\int_1^4 h(x)\,dx = 6$, then $\int_1^4 h(5-x)\,dx =$

a) -6 b) -1 c) 0 d) 3 e) 6

$-\int_1^4 h(5-x)(-dx) = -\int_4^1 h(u)\,du = \int_1^4 h(u)\,du = 6$

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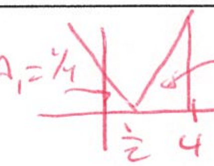
2. $\int_0^4 |2x-1|\;dx$

$A_1 = 1/4$



$A_2 = 12.25$

a) 10 b) 11 c) 11.5 d) 12 e) 12.5

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3. $\int_0^1 \frac{1}{\sqrt{2-t^2}}\,dt = \sin^{-1}\frac{t}{\sqrt{2}}\Big|_0^1 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(0)$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{12}$ e) $\frac{\pi}{8}$

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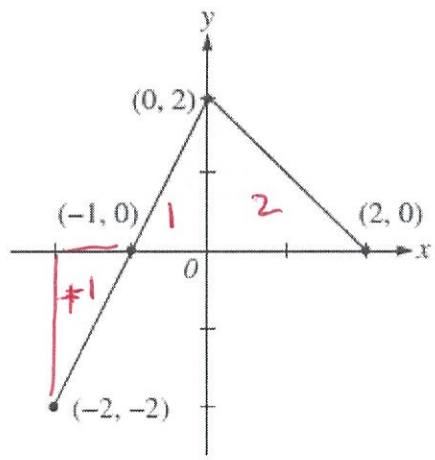
4. Find the average rate of change of $y = x^2 + 5x + 14$ on $x \in [-1, 2]$

a) 3 b) 6 c) 9 d) $\frac{65}{6}$ e) 18

$\frac{f(b) - f(a)}{b - a} = \frac{28 - (-10)}{2 - (-1)}$



Graph of f

5. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x)=\int_{-1}^{x} f(t)\,dt$, then $g(-2)=$ $\int_{-1}^{-2} = -\int_{-2}^{-1}$ $= -(-1)$

a) -2 b) -1 c) 0 d) 1 e) 2

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6. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$? $= \frac{°C}{HR}$ $H' =$ INSTANTANEOUS RATE OF CHANGE

a) The change in temperature during the first day.
b) The change in temperature during the 24th hour.
c) The average rate at which the temperature changed during the 24th hour.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24th hour.

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7. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Midpoint Riemann Sum is used to approximate $\int_1^5 f(x)\,dx$ the result is

a) 3.7 b) 4.5 c) 4.6 d) 5.2

e) none of these

$= 2(1.1) + 2(1.2) = 4.6$

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CALCULATOR ALLOWED
Directions: Show all work.
Do not use math 9.

$du=3dx$ $\quad u=4+x^2$ $\quad \frac{du}{2}=2xdx$

$u=3x$

1. $\int_0^{\pi/9}\left(\tan 3x + \frac{x}{4+x^2}\right)dx$. Show the anti-derivatives.

$$\frac{1}{3}\ln|\sec u|\Big|_0^{\pi/3} + \frac{1}{2}\ln|4+x^2|\Big]_0^{\pi/9} = .246$$

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2. Find the average value of $y=\frac{4}{x}\ln^3 x$ on $x\in[1,\ e]$. Show the anti-derivative.

$$\frac{1}{e-1}\int_1^e \frac{4}{x}\ln^3 x = \frac{4}{e-1}\int_0^1 u^3\,du$$

$$= \frac{4}{e-1}\left[\frac{u^4}{4}\right]_0^1 = \frac{1}{e-1} = .582$$

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3. Find the area between $y = xe^{3x^2}$ and the x-axis on $x \in [-2, 1]$. Show the anti-derivative.

$$A = -\int_{-2}^{0} xe^{3x^2} dx + \int_{0}^{1} xe^{3x^2} dx$$

$$= -\frac{1}{6}\int_{12}^{0} e^{u} + \frac{1}{6}\int_{0}^{3} e^{u} du$$

$$= 27128.812$$

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4. Find the area on $x \in [0, 2]$ under $f(x) = \frac{1}{x^2+9} + \sin 4x$. Show the anti-derivative.

$$A = \int_{0}^{.811} f(x) dx - \int_{.811}^{1.547} f(x) dx + \int_{1.547}^{2} f(x) dx$$

$$= \left[\frac{1}{3} \tan^{-1}\frac{x}{3} - \frac{1}{4}\cos 4x\right] - [\;--\;] + ----$$

$$= 1.336$$

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5. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + .8t\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

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a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

2

$\int_0^2 f(t)\,dt = 20.051$ LBS

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b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

$f'(7) = -8.120$ MEANS LBS/HOUR/HOUR IS THE RATE OF CHANGE OF ~~ARE BEING REMOVED FROM THE TABLE AT~~ $t = 7$ HRS

(2) AFTER THE STORE IS OPEN 7 HOURS, THE RATE AT WHICH BANANAS ARE BEING REMOVED IS DECREASING BY 8,120 LBS PER HOUR PER HOUR

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c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

$g(5) - f(5) =$

$11,532 - 13,796 = -2.264 \frac{\text{LBS}}{\text{HR}}$ DECREASING BECAUSE < 0

(2)

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d) How many pounds of bananas are on the display table at time $t = 8$?

(3) $A = 50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt$

$= \cancel{41.844}$ LBS

$= 23.347$ LBS

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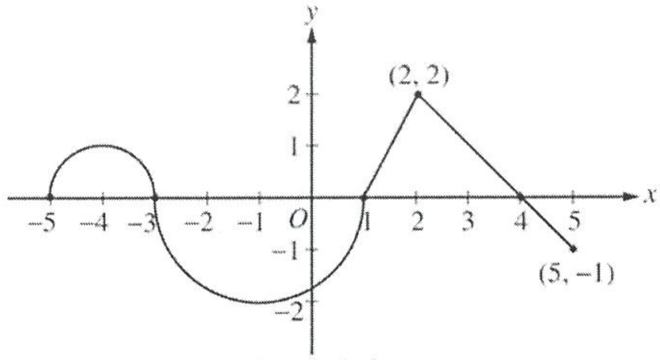
6. The graph of $f'(x)$, defined on $x \in [-5, 5]$, is shown below. $f'(x)$ consists of two semi-circles and two line segments.



Graph of f'

Also, let $f(2) = 3$.

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a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

$x = -3, 4$ BECAUSE f' SWITCHES FROM + TO −

3

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b) Write the equation of the line tangent to $f(x)$ at $x = 2$.

1

$y - 3 = 2(x - 2)$

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c) On what interval(s), if any, is the graph of $f(x)$ both increasing and concave up? Justify your answer.

$f' > 0$ f' IS INCREASING

$x \in (-5, -4) \cup (1, 2)$

2

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d) Find the absolute maximum value of $f(x)$ on $x \in [-5, 5]$. Justify your answer.

3

MAXIMUMS @ $x = -3$ & 4

$f(-3) = 3 + \int_2^{-3} f'(t)\,dt = \boxed{\frac{5}{2} + 2\pi}$

$f(4) = 3 + \int_2^4 f'(t)\,dt = 5$
