AP Calculus BC '19
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Derivative Test

Name: SOLUTION KEY

Multiple choice – Circle correct answer.

1. Let f be a differentiable function with $f(3)=2$ and $f'(3)=5$, and let g be a function defined by $g(x)=xf(x)$. Which of the following is an equation of the line normal to the graph of g at the point where $x=3$?

a) $y-2=5(x-3)$

b) $y-2=-\frac{1}{5}(x-3)$

~~$y_1 = 2$ for x_1~~ $= 3$

c) $y-6=17(x-3)$

d) $y-6=\frac{1}{17}(x-3)$

e) $y-6=-\frac{1}{17}(x-3)$

$g(3) = 3f(3) = 6$

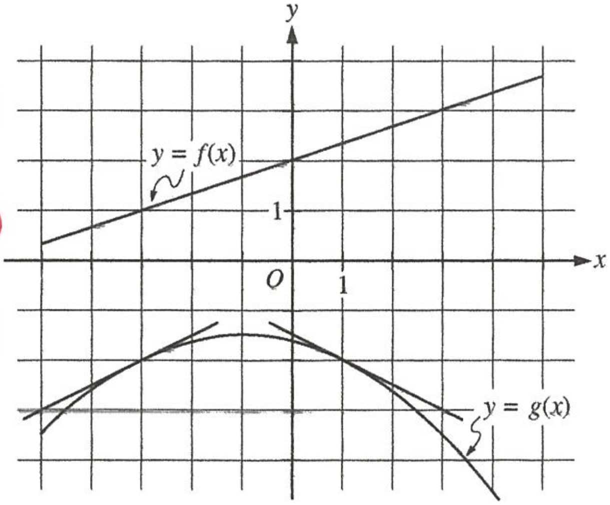
$f' = 5$ BUT $g' = x(f'(x)) + f(x)(1)$

$g'(3) = 3(f'(3)) + 2 = 17$

$m_{TAN} = 17$

$m_{NORMAL} = \frac{-1}{17}$

2. The figure below shows the graph of the functions f and g. The graphs of the lines tangent to the graph of g at $x=-3$ and $x=1$ are also shown. If $B(x)=f(g(x))$, what is $B'(1)$?



a) $-\frac{1}{2}$

b) $-\frac{1}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{3}$

e) $\frac{1}{2}$

$B' = f'(g(x))\cdot g'(x)$

$B'(1) = f'(g(1))\cdot g'(1)$

$= f'(2)\cdot g'(1)$

$= \frac{1}{3}\left(-\frac{1}{2}\right)$

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3. Which of the following statements must be true?

I. $\frac{d}{dx}\left(x\sec^{-1}x\right)=\sec^{-1}x+\frac{1}{\sqrt{x^2-1}}$

II. $\frac{d}{dx}\left(\frac{3-2x}{3x+2}\right)=\frac{13}{(3x+2)^2}$

III. $\frac{d}{dx}\ln(1-x)=\frac{1}{x-1}$

a) I only b) II only c) II and III only

d) I and III only e) I, II, and III

I $x\left(\frac{1}{x\sqrt{x^2-1}}\right)+\sec^{-1}x\,(1)$

II $\frac{(3x+2)(-2)-(3-2x)(3)}{(3x+2)^2}=\frac{-13}{(3x+2)^2}$

III $\frac{1}{1-x}(-1)=\frac{1}{x-1}$

4. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y + xy$ with the initial condition $f(0) = 1$. What is the best approximation for $f(3)$ if Euler's method is used, starting at $x = 0$ with a step size of 1?

a) 0 b) 1 c) 2 d) 6 e) 24

POINT	M	TANGENT LINE	NEW X	NEW Y
(0,1)	1	$y-1=x$	1	2
(1,2)	4	$y-2=4(x-1)$	2	6
(2,6	18	$y-6=18(x-2)$	3	24

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5. If $f(x) = \sin[g(x)]$, then $\frac{d}{dx}[f(x)]$ is

a) $\sin x \cdot g'(x) + g(x) \cdot \cos x$

b) $\sin x \cdot g'(x) - g(x) \cdot \cos x$

c) $\cos x \cdot g'(x)$

d) $\cos[g(x)] \cdot g'(x)$

e) None of these

$f'(x) = \cos[g(x)] \cdot g'(x)$

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6. The slope of the line tangent to the curve $xy^3 + x^2y^2 = 6$ at $(2,1)$ is

a) -1 b) $-\frac{3}{5}$ c) $-\frac{5}{14}$ d) $-\frac{3}{14}$ e) 0

$$x\left(3y^2\frac{dy}{dx}\right) + y^3(1) + x^2\left(2y\frac{dy}{dx}\right) + y^2(2x) = 0$$

$$2(3)\frac{dy}{dx} + 1 + 4\left(2\frac{dy}{dx}\right) + 4 = 0$$

$$14\frac{dy}{dx} = -5$$

FREE RESPONSE – show all work in a clear, organized manner. Simplify answers.

7. $\frac{d}{dx}\left[-3x^4 + 7 - \frac{6}{5}x^{5/3} - \frac{3}{\sqrt[5]{x^6}} - \frac{1}{12x}\right] = \frac{d}{dx}\left[-3x^4 + 7 - \frac{6}{5}x^{5/3} - 3x^{-6/5} - \frac{1}{12}x^{-1}\right]$

$$= -12x^3 - 2x^{2/3} + \frac{18}{5}x^{-11/5} + \frac{1}{12}x^{-2}$$

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8. If $g(x) = \sec^{-1} 3x^2$, find $g''(x)$

$$g'(x) = \frac{1}{3x^2\sqrt{9x^4-1}}(6x) = \frac{2}{x\sqrt{9x^4-1}} = 2x^{-1}(9x^4-1)^{-1/2}$$

$$g''(x) = +2x^{-1}\left(-\frac{1}{2}(9x^4-1)^{-3/2}(36x^3)\right) + (9x^4-1)^{-1/2}(-2x^{-2})$$

$$= \frac{-36x^2}{(9x^4-1)^{3/2}} - \frac{2}{x^2(9x^4-1)^{1/2}}$$

$$= \frac{-36x^4 - 2(9x^4-1)}{x^2(9x^4-1)^{3/2}} = \frac{-54x^4+2}{x^2(9x^4-1)^{3/2}}$$

9. If $f(x) = e^{\cot 3x}$, find $f''(x)$.

$$f'(x) = e^{\cot 3x}(-\csc^2 3x)(3) = -3e^{\cot 3x}(\csc^2 3x)$$

$$f''(x) = -3e^{\cot 3x}\left(2\csc 3x(-\csc 3x\cot 3x)(3)\right) + \csc^2 3x\left(-3e^{\cot 3x}\cdot(-3\csc^2 3x)\right)$$

$$= 18e^{\cot 3x}\csc^2 3x\cot 3x + 9\csc^4 3x\, e^{\cot 3x}$$

$$= 9e^{\cot 3x}\csc^2 3x(2\cot 3x + \csc^2 3x)$$

10. Given $x^3 + xy = 2y^2 + y + 7$.

a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2 + y}{4y - x + 1}$

$$3x^2 + x\frac{dy}{dx} + y(1) = 4y\frac{dy}{dx} + \frac{dy}{dx}$$

$$3x^2 + y = (4y - x + 1)\frac{dy}{dy}$$

$$\frac{dy}{dx} = \frac{3x + y}{4y - x + 1}$$

b) Find the slopes of the tangent lines at all the points where the curve intersects the line $x = 2$.

$$x = 2 \rightarrow 8 + 2y = 2y^2 + y + 7$$

$$0 = 2y^2 - y - 1 = (2y+1)(y-1)$$

$$y = -1/2, 1$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{12+1}{4-2+1} = \frac{13}{3} \qquad \left.\frac{dy}{dx}\right|_{(2,-1/2)} = \frac{12 - 1/2}{-2-2+1} = -\frac{23}{6}$$

c) Set up, but do not solve, an equation to determine all the points on the curve that have a vertical tangent line.

$$4y - x + 1 = 0 \longrightarrow x = 4y - 1$$

$$(4y-1)^3 + (4y-1)y = 2y^2 + y + 7$$