AP Calculus BC ’18-19

Fall Final
Part IIa

Calculator Required

Name:

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1. At time $t = 0$, there are 120 gallons of oil in a tank. During the time interval $0 \le t \le 10$ hours, oil flows into the tank at a rate of $h(t) = 10 - \frac{t\cos(t)}{2}$ and out of the tank at a rate given by $g(t) = 6 + \frac{e^{0.52t}}{t+1}$. Both h and g are measured in gallons per hour.

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a) How much oil flows out of the tank during this 10-hour time period?

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b) Find the value of $h(4.3) - g(4.3)$. Using correct units, explain what this value represents in the context of this problem.

c) Write an expression for $A(t)$, the total amount of oil in the tank at time *t*.

d) Find the absolute maximum and minimum amount of oil in the tank during $0 \leq t \leq 10$ hours.

2. A tree trunk has circular horizontal cross sections. The radius $r(h)$ of the tree trunk is a differentiable function of the height h of the tree measured from the ground. Both r and h are measured in feet. Selected values of r and h are given in the table below.

h height from the ground (feet)	0	1	3	5	8
$r(h)$ radius of the tree trunk (feet)	3	2.5	2	2.5	1.5

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a) Use a Right Riemann sum with subintervals indicated by the table to approximate the value of $\frac{1}{8}\int_0^8 r(h)\,dh$. Using correct units, explain the meaning of this value in the context of this problem.

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b) Must there be a height within the first eight feet of tree trunk where $\frac{dr}{dh} = 0$? Explain.

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c) Write an expression involving one or more integrals to calculate the volume of the tree trunk from $h=0$ to $h=8$. Use a Left Riemann sum to approximate the value of this expression.

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d) For heights above 8 feet, the radius is given by $g(h)=\frac{1}{h^2}+\frac{95}{64}$. A squirrel climbs up the tree at a rate of $\frac{dh}{dt}=3\ \text{ft}/\text{sec}$. How quickly is the radius of the tree changing when the squirrel is 9 feet above the ground?

End of

AP Calculus BC ’18-19

Fall Final -- Part IIa

AP Calculus BC ’18-19

Fall Final
Part IIb

No Calculator Allowed

Name:

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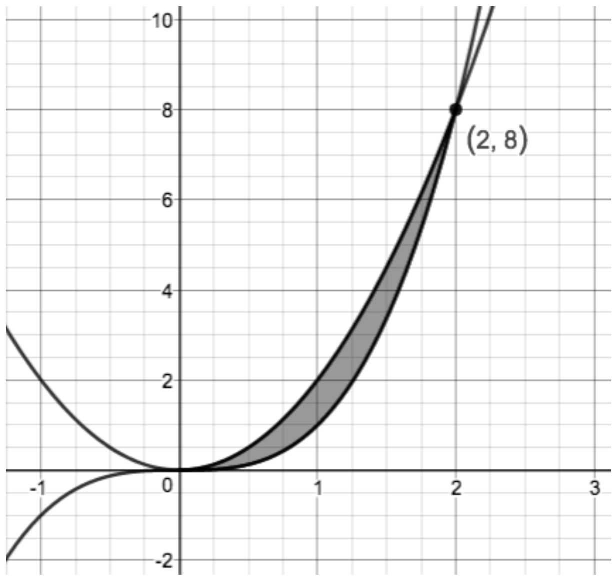
3. Let R be the region bounded by $y = 2x^2$ and $y = x^3$, shaded in the picture below. The curves intersect at the origin and at the point $(2,8)$.



a) Find the area of R.

b) Set up, but do not evaluate, an expression involving one or more integrals that would find the volume of the solid whose base is *R* and whose cross sections parallel to the y-axis are semicircles.

c) Find the volume of the solid formed by revolving *R* around the y-axis.

4. The temperature of water in a pond is modeled by the equation $B(t)$, which satisfies the differential equation $\frac{dB}{dt} = -\frac{\sqrt{B}}{8} + 1$. At time $t = 0$ hours, the pond's temperature is 81°F.

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a) Write the equation of the line tangent to B at $t = 0$ and use it to approximate the temperature of the pond at $t = 1$.

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b) For what value(s) of B does $B(t)$ have a critical value?

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c) Find $\frac{d^2B}{dt^2}$ in terms of B.

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d) A researcher suggests a different possible model for the temperature of the lake as $P(t)$, where $\frac{dP}{dt} = \frac{-10}{(2t+1)^3}$. The pond still begins with a temperature of 81°F at $t = 0$. Using this model, what temperature would the pond converge to after an infinite amount of time?

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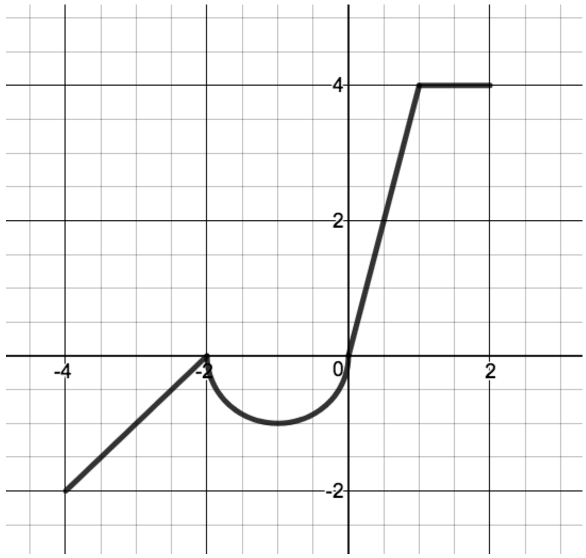
5. Below is the graph of $g(x)$, which is defined on $-4 \le x \le 2$ and consists of three line segments and a semicircle. Let $f(x) = \int_1^x g(t)\,dt$.



graph of $g(x)$

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a) Find $f(-2)$, $f'(-2)$, and $f''(-2)$

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b) On what intervals, if any, is f(x) both decreasing and concave up? Justify your answer.

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c) Find the average rate of change of $f(x)$ from $x=-2$ to $x=2$.

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d) Identify all x-values in the open interval $-4 < x < 2$ at which $f(x)$ has a critical point, and classify each critical point as a local minimum, local maximum, or neither. Justify your answers.

6. A twice-differentiable function $y = f(x)$ has derivative given by the differential equation $\frac{dy}{dx} = \frac{x(y^2-1)}{y}$ and satisfies $f(1) = -\sqrt{2}$.

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a) Write the equation of the line **normal** to $y = f(x)$ at the point $(1, -\sqrt{2})$.

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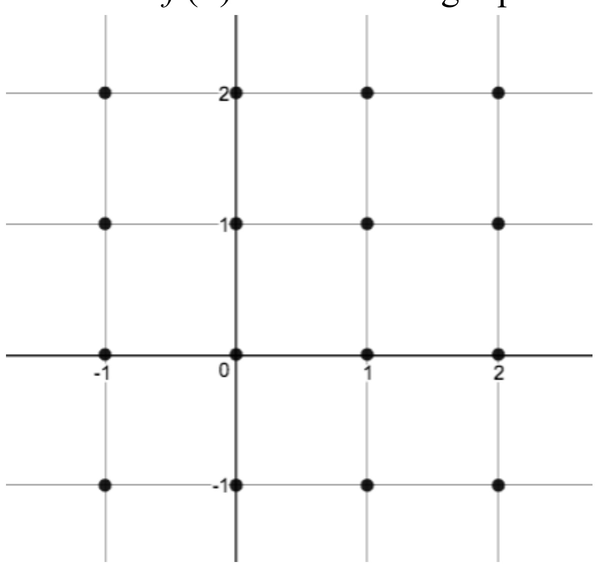
b) Sketch the slope field for $f(x)$ at the 16 integer points indicated.



c) Find the particular solution $y = f(x)$ for which $f(1) = -\sqrt{2}$.

End of

AP Calculus BC ’18-19

Fall Final