BC Calculus 2018-19 Name Solution Key

Limit and Continuity Test

NO CALCULATOR ALLOWED Score______

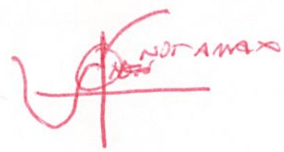
1. Let $f(x)=\begin{cases} \cos x, \ if \ x\le 0 \\ 1+\sqrt{x}, \ if \ 0<x \end{cases}$. Which of the following statements is true about f?

$f' = \begin{cases} -\sin x & x<0 \\ -\frac{1}{2\sqrt{x}} & \text{if } 0<x \end{cases}$

T I. f is continuous at $x = 0$.

F ~~II.~~ f is differentiable at $x = 0$. $\sin 0 \neq -\frac{1}{2\sqrt{0}}$

F ~~III.~~ f has a local maximum at $x = 0$.



a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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2. The function f is not differentiable at $x = b$. Which of the following statements **must** be true?

(a) $\lim_{x\to b} f(x)$ dne (b) $\lim_{x\to b} f(x) \neq f(b)$ (c) $\lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x)$

(d) $\lim_{x\to b^-} f'(x) \neq \lim_{x\to b^+} f'(x)$ (e) None of these

Any answer might be true, but none must be true

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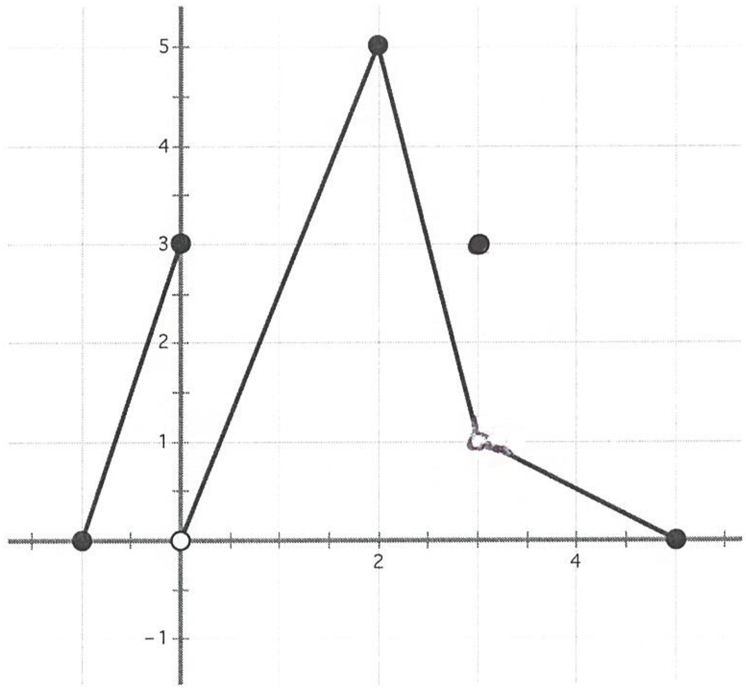
3. The function f is defined on the interval $x \in [-1, 5]$ and has the graph shown below.



Which of the following is (are) false?

I. $\lim_{x \to 2} f(x) = 2$ F Lim = 5

II. $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = dne$ TRUE $\lim_{x \to 3} \frac{f(x)-f(3)}{x-3} = f'(3)$

III. $\lim_{x \to 0^-} f(x) = f(3)$ T

a) I only b) II only c) III only

d) I and II only e) I and III only

4. $\lim_{h\to 0}\dfrac{\ln\left(\dfrac{2+h}{2}\right)}{h}=$ $\dfrac{d}{dx}\ln x\Big|_{x=2}=\dfrac{1}{2}$

(a) e^2 (b) 1 (c) $\dfrac{1}{2}$ (d) 0 (e) DNE

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5. $\lim_{x\to\infty}\dfrac{4x^5+3x^4+2x^3+x^2+1}{3x^6-9x^4+4x^3+15}=$ 0 DENOM DEG > NUM DEGREE

(a) 0 (b) $\dfrac{3}{4}$ (c) $\dfrac{4}{3}$ (d) 3 (e) DNE

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6. Which of the following improper integrals diverge?

I. $\displaystyle\int_0^{\infty}\frac{2x}{1+x^2}\,dx$ $=\frac{1}{2}\ln x\Big]_0^{\infty}$ D

~~II.~~ $\displaystyle\int_0^{9}\frac{1}{\sqrt{x}}\,dx$ $2\sqrt{x}\Big]_0^{9}$ CON

III. $\displaystyle\int_1^{\infty}\frac{1}{\sqrt{x}}\,dx$ $2\sqrt{x}\Big]_1^{\infty}$ D

(a) I only (b) II only (c) I and II only

(d) II and III only (e) I and III only

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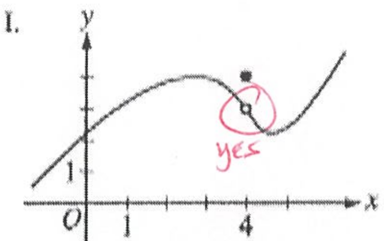
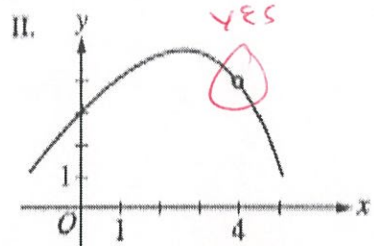
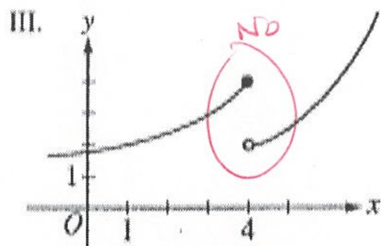
7. For which of the following does $\lim_{x\to 4} f(x)$ exist?

I.


Graph of f

II.


Graph of f

III.


Graph of f

(a) I only (b) II only (c) III only (d) I and II only (e) I and III only

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8. Let $f(x)=\begin{cases} \sin^{-1}(1-x), & \text{if } x<1 \\ 0, & \text{if } x=1 \\ \ln x^2, & \text{if } x>1 \end{cases}$. Which of the following statements is true about f?

$f' = \begin{cases} \dfrac{-1}{\sqrt{1-(1-x)^2}} \\ \dfrac{2}{x} \end{cases}$

I. f is continuous at $x = 1$. $\sin^{-1}(1-1)=0=\ln 1^2$

~~II.~~ f is differentiable at $x = 1$. $f'(1^-) \neq f'(1^+)$

III. f has a local minimum at $x = 1$.

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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9. Which of the following functions is NOT differentiable at $x=0$?

(a) $f(x)=x^2$ (b) $f(x)=e^x$ (c) $f(x)=\ln(x+1)$

(d) $f(x)=\begin{cases} \dfrac{1}{x+1} & \text{for } x\neq -1 \\ 0 & \text{for } x=-1 \end{cases}$ (e) $f(x)=\cot x$

VA @ $x=0$

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10. Let $F(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x does $F(x)$ attain a minimum?

(a) No values of x (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2 (e) 3

$F' = e^{(x^2-3x)^2}(2x-3)=0 \quad x=3/2$

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11. The function f is defined for all Real numbers such that

$$f(x)=\begin{cases} x^2+kx & \text{for } x<3 \\ 5\sin\frac{\pi}{2}x & \text{for } x\geq 3 \end{cases}.$$

$9+3k=-5$

$k=-14/3$

For which value of k will the function be continuous throughout its domain?

(a) -2 (b) -1 (c) $\frac{2}{3}$ (d) 1 (e) None of these

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12. Given $f(x)=\frac{1}{\sqrt{x}}$, $\int_0^{\infty} f(x)\,dx$ is

$\lim_{x \to 0^+} 2\sqrt{x} = 0$

$\lim_{x \to \infty} 2\sqrt{x} = \text{DNE}$

a) convergent.

b) divergent because $\lim_{x\to 0^+} f(x)$ does not exist.

c) divergent because $\lim_{x\to\infty} f(x)$ does not exist.

d) divergent because neither $\lim_{x\to 0^+} f(x)$ nor $\lim_{x\to\infty} f(x)$ exist.

e) none of the above.

AP Calculus BC 2018-19
Limit/Continuity Test
NO CALCULATOR ALLOWED

Name SOLUTION KEY

Score______

1. $f(x)=\begin{cases} \sin(1-x), \text{ if } x<1 \\ 0 \ , \quad \text{if } x=1 \\ \ln\frac{1}{x}, \text{ if } 1<x \end{cases}$

a) Is $f(x)$ continuous? Why/Why not?

i) $f(1)$ EXISTS

ii) $\lim_{x\to 1^-} f(x) = \sin 0 = 0 = \ln 1 = \lim_{x\to 1^+} f(x)$

iii) $\lim_{x\to 1} f(x) = f(1)$

$\therefore f(x)$ IS CONTINUOUS

b) Is $f(x)$ differentiable? Why/Why not?

$f'(x) = \begin{cases} \cos(1-x)(-1) & \text{IF } x<1 \\ \frac{1}{1/x}\left(\frac{-1}{x^2}\right) = -\frac{1}{x} & \text{IF } x>1 \end{cases}$

i) $f(x)$ IS CONTINUOUS

ii) $f'(x)$ EXISTS FROM BOTH THE LEFT & RIGHT OF 1

iii) $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1} [-\cos(1-x)] = -1 = \lim_{x\to 1} \frac{-1}{x} = \lim_{x\to 1^+} f(x)$

$\therefore f(x)$ IS DIFFERENTIABLE

$u = -x^2 \quad du = -2x\,dx$

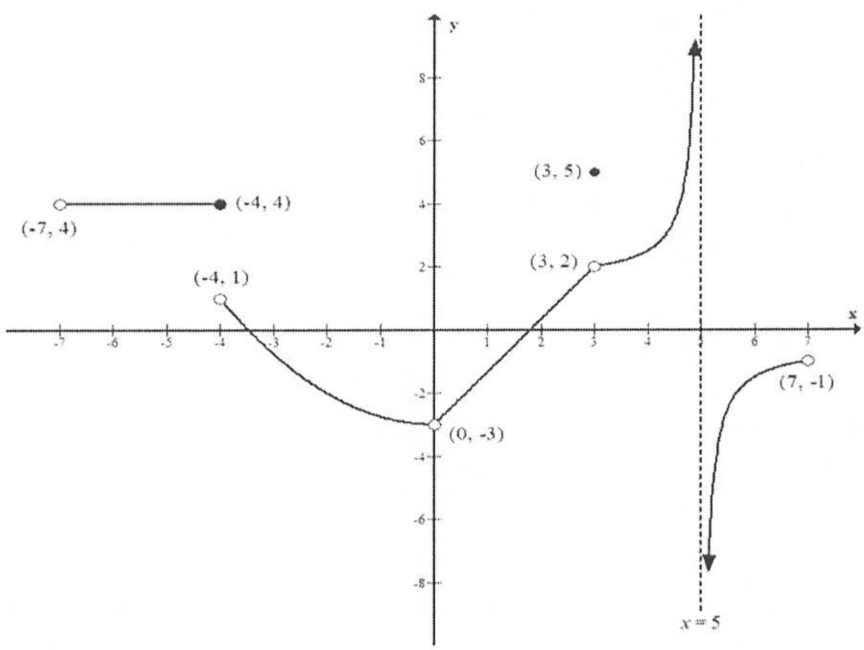
2. Evaluate $\int_{-\infty}^{0} xe^{-x^2}\,dx$

$= \lim_{a \to -\infty} \int_a^0 xe^{-x^2}\,dx = \lim_{a \to -\infty} -\frac{1}{2}\int_a^0 e^u\,du$

$-\frac{1}{2}\lim_{a \to -\infty} e^u \Big]_{-a^2}^{0} = -\frac{1}{2}e^0 - \lim_{a \to -\infty} e^{-a^2}$

$= -\frac{1}{2} - 0$

$= -\frac{1}{2}$



3. For this graph, find

(a) $\lim_{x \to -4^-} f(x) = 4$ (b) $\lim_{x \to -4^+} f(x) = 1$ (c) $\lim_{x \to 3} f(x) = 2$

(d) $\lim_{x \to -4} f(x) =$ ~~4~~ DNE (e) $\lim_{x \to 5^+} f(x) = -\infty$ (f) $\lim_{x \to 5^-} f(x) = +\infty$

(g) $f(-4) = 4$ (h) $\lim_{x \to 0} f(x) = -3$ (i) $f(0) =$ DNE (j) $f(3) = 5$

4. $$\int_{-2}^{0} \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= \lim_{a \to -2^+} \int_a^0 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= \lim_{a \to -2^+} \sin^{-1} \frac{x}{2} \Big]_a^0$$

$$= \sin^{-1} 0 - \lim_{x \to -2^+} \sin^{-1}\left(\frac{a}{2}\right)$$

$$= \sin^{-1} 0 - \sin^{-1}(-1)$$

$$= 0 - \left(-\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}$$