BC Calculus '18-19 Name: Solution Key

Parametric/Polar Test

1. The velocity of a particle's motion is described by $\langle t^2+2t,\ 2t^3+3t^2 \rangle$. At $t=1$, the particle's position is $(3,-5)$. $x(-1) = 3 + \int_1^{-1} (t^2+2t)\,dt = 7/3$

a) -0.667 b) 2.333 c) -5.667 d) 3.667 e) -1

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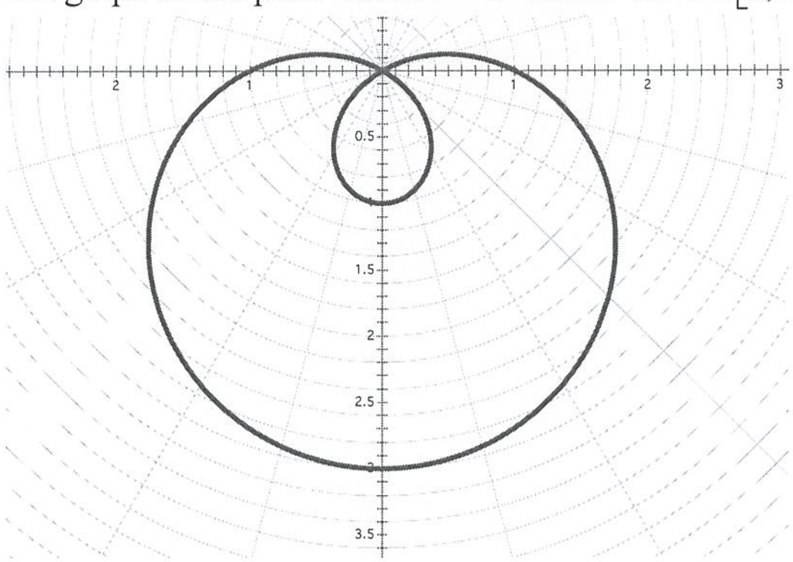
2. Below is the graph of the polar curve $r = 1 - 2\sin\theta$ on $\theta \in [0, 2\pi]$.



$0 = 1 - 2\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \begin{cases} \pi/6 \pm 2\pi n \\ 5\pi/6 \pm 2\pi n \end{cases}$

What are the boundaries for the inner loop of the curve?

a) $\theta \in [0, 2\pi]$ b) $\theta \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ c) $\theta \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$

d) $\theta \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$ e) $\theta \in \left[-\frac{11\pi}{6}, \frac{\pi}{6}\right]$

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3. A curve is described by the parametric equations $x = 3t^2 - 19t$ and $y = e^{2t-7}$ Find the slope of the line tangent to the path of the curve at $t = 4$.

a) $-\frac{e}{28}$ b) $-\frac{28}{e}$ c) $\frac{e}{5}$ d) $\frac{2e}{5}$ e) $\frac{5}{2e}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{2e^{2t-7}}{6t-19}\right|_{x=4} = \frac{2e^1}{5}$$

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4. If $x(t) = 5\cos t$ and $y(t) = 3\sin t$, then $\frac{d^2y}{dx^2} =$

a) $-\frac{3}{5}\cot t$ b) $\frac{3}{5}\tan t$ c) $\frac{3}{5}\csc^2 t$

d) $\frac{3}{25}\csc^2 t$ e) $-\frac{3}{25}\csc^3 t$

$$\frac{dy}{dt} = \frac{3\cos t}{-5\sin t} = -\frac{3}{5}\cot t$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{5}(-\csc^2 t)}{-5\sin t} = \frac{-3}{25}\csc^3 t$$

$3\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}$

$\theta = \pi/6$

5. The total area enclosed by $r = 3\cos 3\theta$ is

a) $\frac{7\pi}{4}$ b) 2π c) $\frac{9\pi}{4}$ d) $\frac{5\pi}{2}$ e) $\frac{11\pi}{4}$

$$A = 3\int_0^{\pi/6} 2\left(\frac{1}{2}(3\cos 3\theta)^2\right) d\theta =$$

$$9 \cancel{27} \int_0^{\pi/6} \cos^2 3\theta \, d\theta = \overset{3}{\cancel{9}}\left[\frac{1}{2}\theta + \frac{1}{4}\sin 6\theta\right]_0^{\pi/6}$$

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6. A curve's velocity is described by the parametric equations $x'(t) = t^2 + 2t$ and $y'(t) = t^3 + t^2$. The length of this curve from t=0 to t=2 is:

a) 14.422 b) 9.603 9.~~063~~ c) 13.333

d) 14.941 e) 20

$$L = \int_0^2 \sqrt{(t^2+2t)^2 + (t^3+t^2)^2}$$

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7. A particle moves on a plane so that its position vector is

$$p(t)=\left\langle \frac{1}{3}t^3+\frac{1}{2}t^2-2t+7,\ \frac{2}{3}t^3+\frac{3}{2}t^2-2t+\pi^6 \right\rangle \text{ is at rest when}$$

a) $t=1$ only

b) $t=\frac{1}{2}$ only

c) $t=-2$ only

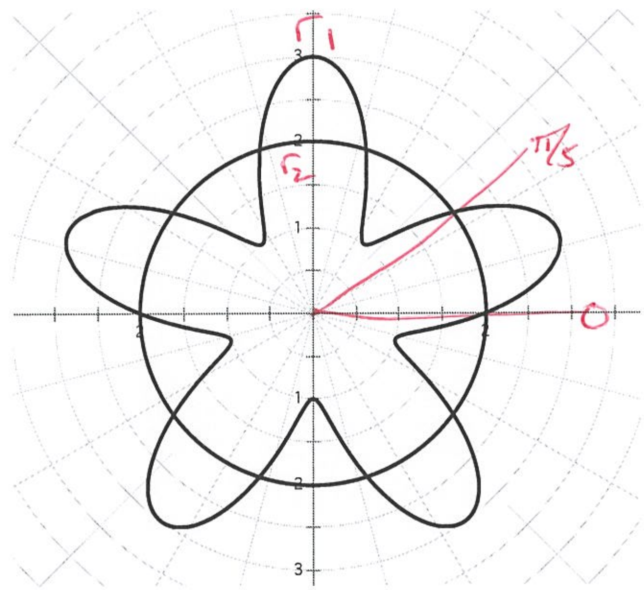
d) $t=1,\ \frac{1}{2}$

e) $t=1,\ \frac{1}{2},\ -2$

$V=\langle t^2+t-2,\ \cancel{3t-2}\ 2t^2+3t-2\rangle$

$=\langle (t+2)(t-1),\ (2t-1)(t+2)\rangle$

$t=-2$



$2+\sin 5\theta=2$

$\sin 5\theta=0$

$5\theta=0\pm\pi n$

$\theta=0\pm\frac{\pi}{5}n$

8. The graphs of $r_1=2+\sin(5\theta)$ and $r_2=2$ are shown above. What is the total area inside r_1 and outside r_2?

a) 1.571 b) 1.000 c) 3.142 d) 3.125 e) 4.785

$A=\frac{5}{2}\int_0^{\pi/5}(2+\sin 5\theta)^2-2^2\,d\theta=$

9. Given the equation $r = \theta + \sin 2\theta$, which of the following would be the derivative needed to determine the rate of change of the distance from the pole to the curve, relative to the angle, at a given point?

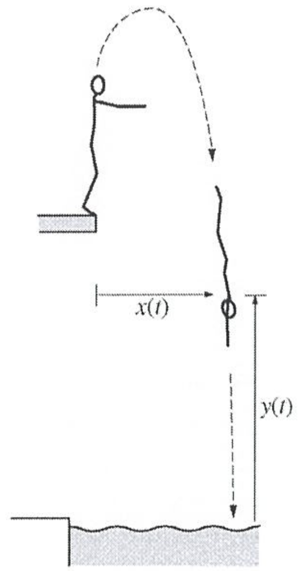
a) $\dfrac{dy}{dx}$ b) $\dfrac{d\theta}{dr}$ c) $\dfrac{d\theta}{dt}$ d) $\dfrac{dr}{dt}$ e) $\dfrac{dr}{d\theta}$

10.

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

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a. Find the maximum vertical distance from the water surface to the diver's shoulders.

3

$$\frac{dy}{dt} = 0 \rightarrow t = .36735$$

$$y(.367) = 11.4 + \int_0^{.36735} (3.6 - 9.8t)\,dt = 12.061$$

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b. Find A, the time that the diver's shoulders enter the water.

2

$$y(A) = 0 = 11.4 + \int_0^{A} \frac{dy}{dt} \Rightarrow 11.4 + 3.6A - 4.9A^2 = 0$$

$$A = 1.936\text{sec}$$

c. Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(2) $\int_0^{1.936} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 12.946$ m

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d. Find the angle $\theta \in \left[0, \pi/2\right]$ between the path of the diver and the water at the instant the diver's shoulders enter the water.

$\text{TAN}\ \theta = \frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1.936} = -19.21913$

(2)

$\theta = \text{TAN}^{-1} -19.219 = 1.518$ OR 1.549

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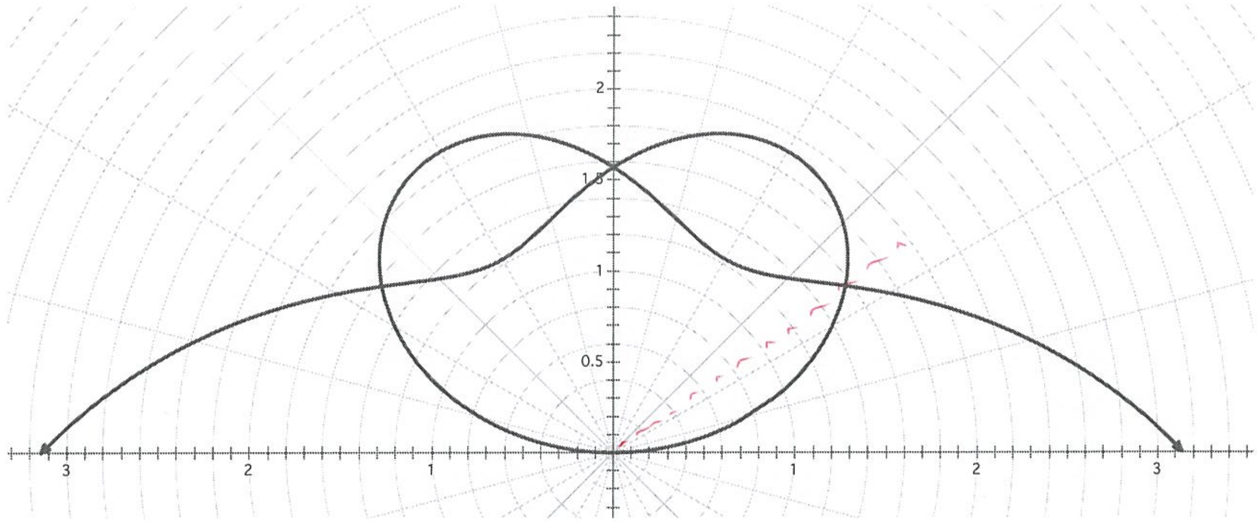
11. The graph below is $r=\theta+\sin 2\theta$ on $\theta\in[-\pi,\pi]$.



a. Find the length of the curve.

(3)

$$L=\int_{-\pi}^{\pi}\sqrt{(\theta+\sin 2\theta)^2+(1+2\cos 2\theta)^2}$$

$$= 14.339$$

b. Assume the point where the curve intersects itself in QI has $\theta = -2.487$. Find the area of the region enclosed by $r = \theta + \sin 2\theta$ from $\theta = -\pi$ to the point of intersection.

$$A = \int_{-\pi}^{-2.487} \frac{1}{2}(\theta + \sin 2\theta)^2 d\theta - \frac{1}{2}\int_0^{.655} (\theta + \sin 2\theta)^2 d\theta$$

4

$$= 1.383$$

c. Find the formula for the slope of the line tangent to $r = \theta + \sin 2\theta$ in terms of θ.

$$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(\theta + \sin 2\theta)\sin\theta}{\frac{d}{d\theta}(\theta + \sin 2\theta)\cos\theta}$$

2

$$= \frac{(\theta + \sin 2\theta)\cos\theta + \sin\theta(1 + 2\cos 2\theta)}{(\theta + \sin 2\theta)(-\sin\theta) + \cos\theta(1 + 2\cos 2\theta)}$$