AP Calculus BC ’18-19

Spring Final Part IIA

Calculator Allowed

Name:

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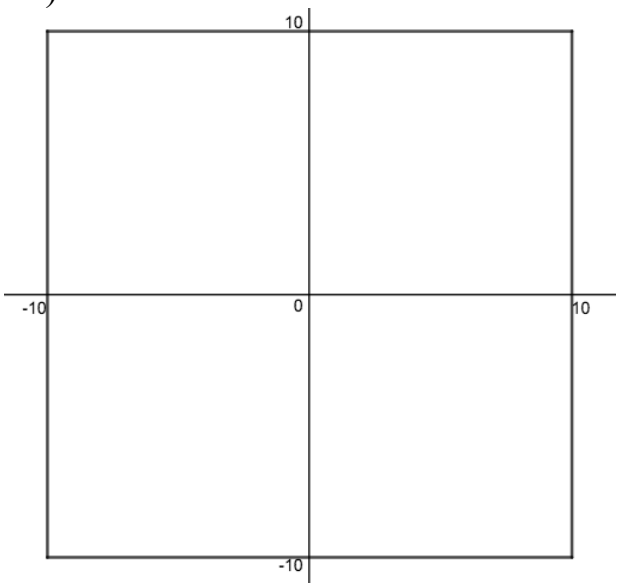
1. A dog named Sparky runs around in his owner's 20 ft x 20 ft backyard. Sparky's owner is a mathematician and has laid an xy-coordinate grid in his backyard, with the origin at the center of the yard. Sparky's position at time t seconds can be modeled in feet by the parametric vector

$$\langle x(t), y(t) \rangle = \langle t\sin(2t), t\cos(3t) \rangle \text{ for } 0 \le t \le 10 \text{ seconds.}$$

(a) What is Sparky's speed at $t = 5$ seconds?

(b) How many feet did Sparky run during the first 3 seconds?

(c) On the graph below, draw Sparky's path of motion in the backyard during the time interval $t = 0$ to $t = 10$. (Hint: use parametric mode on your calculator and adjust the time window.)



(d) Sparky is not allowed to run into the flower garden, which is a vertical strip in the yard starting at $x = -7$ and going until the left fence ($x = -10$). Sparky is a rebellious dog and enters the flower garden anyway. For how long is he in the flower garden during his 10-second run?

2. At 6am at the *Popular Potatoes* potato chip factory, there are already 5 tons of potatoes in the factory. More potatoes are delivered from 6am ($t = 6$) until noon ($t = 12$) at a rate modeled by

$$P(t) = 9 - \frac{9\sin(x-2)}{x-2} \text{ tons of potatoes per hour.}$$

Workers arrive at 6am and begin to process the potatoes to turn them into potato chips. Their supervisor measures their rate of output every hour and records her findings in the chart below.

t = time after midnight in hours	6	9	13	14	16
C(t) = Rate of potatoes processed in tons/hour	7.9	6.5	3.9	3.1	1.3

The supervisor notices that the workers' rate of processing decreased throughout the day.

(a) How many tons of potatoes arrive at the *Popular Potatoes* factory between 6am and noon?

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(b) Use a Left Riemann sum with subintervals indicated by the table to approximate $\int_6^{16} C(t)\,dt$. Using correct units, explain the meaning of this value in the context of the problem.

(c) Is your approximation in part (b) an under- or over-approximation? Explain.

(d) The workers end their shift at 4pm. At that time, are there still potatoes in the factory left to process? Explain your reasoning.

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AP Calculus BC ’18-19

Spring Final Part IIB

NO Calculator Allowed

Name:

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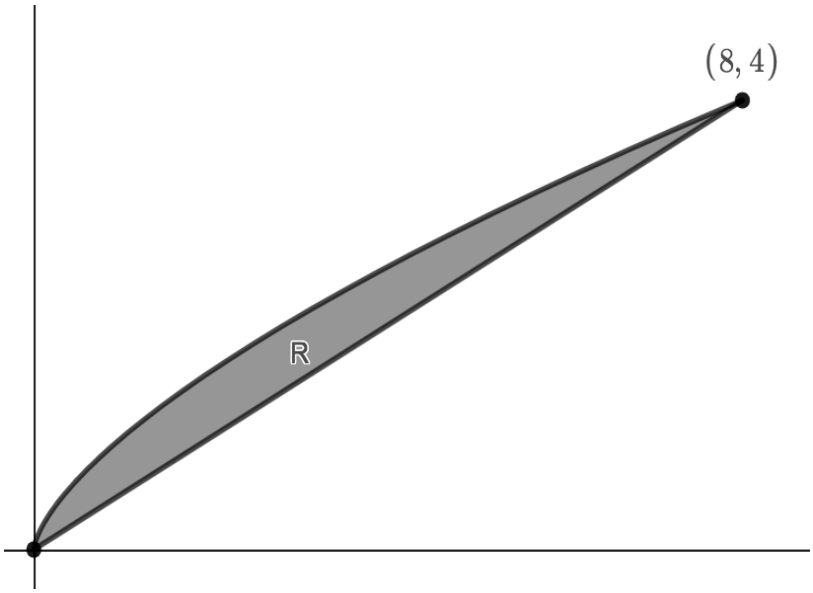
3. Below is the graph of region **R**, which is bounded by the graphs of $f(x)=\frac{x}{2}$ and $g(x)=\sqrt[3]{x^2}$. The curves intersect at the origin and at $(8,4)$.



(a) Find the area of the region **R**.

(b) On the interval $x \in [0,8]$, what x-value gives the greatest vertical distance between the two curves?

(c) A solid **S** is formed by rotating the region **R** around the vertical line $x = -1$. Set up, but do not evaluate, an expression involving one or more integrals that would give the volume of the solid **S.**

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4. The amount of energy E (in Joules) stored in a flux capacitor is modeled as a function of time t in hours. At time $t = 2$, there are 0 Joules of energy in the capacitor. The rate of change of energy in the capacitor is represented by the differential equation

$$\frac{dE}{dt} = \frac{2t+1}{\sec^2 E} \quad \text{Joules/hour}$$

(a) Write the equation for the line tangent to $E(t)$ at $t = 2$ and use it to approximate $E(2.3)$.

(b) Solve for the function $E(t)$ with initial condition $\left(2,0\right)$.

(c) Find $\lim_{t\to\infty} E(t)$

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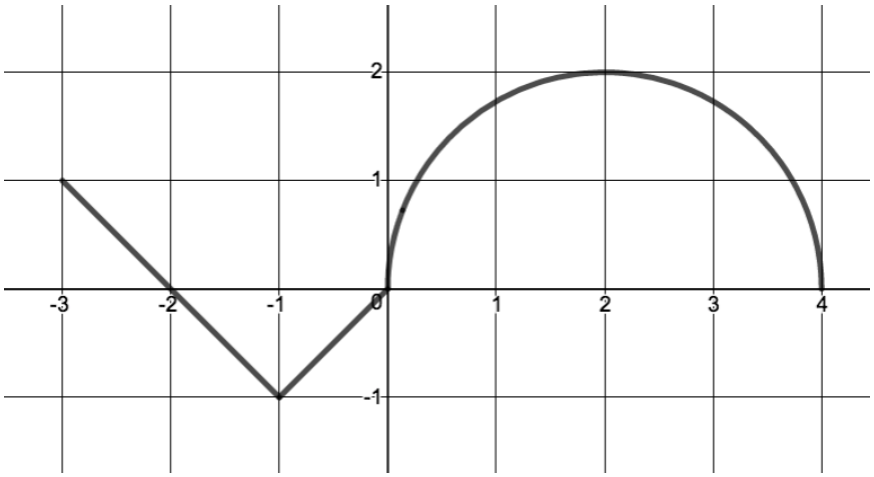
5. The function $h(x)$ is graphed below for the domain $-3 \le x \le 4$. The graph of $h(x)$ consists of line segments and a semi-circle. Let $g(x) = \int_2^x h(t)\,dt$.



graph of $h(x)$

(a) Find $g(4)$ and $g(-3)$.

(b) Find $g'(2)$ and $g''(2)$

(c) Where are the critical points of $g(x)$ on $-3 < x < 4$? Classify them as relative max, min, or neither and explain your reasoning.

(d) What are the absolute maximum and minimum values of $g(x)$ on $-3 \le x \le 4$?

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6. The Taylor series $\sum_{n=0}^{\infty} \frac{3^n}{2^n(n+1)}(x-1)^{n+1} = (x-1) + \frac{3}{4}(x-1)^2 + \frac{3}{4}(x-1)^3 + \cdots$ converges to the function $g(x)$ for all x values such that $|x-1| < R$.

(a) Find $g'(1)$ and $g^{IV}(1)$.

(b) Find the value of R and determine the interval of convergence for this Taylor Series.

(c) Use the Taylor series for $g(x)$ to write the first 3 nonzero terms and the general term for the Taylor series for function $h(x) = x \cdot g'\left(x+1\right)$.

End of

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