AP Calculus BC ‘18-19 Name________________________________
Taylor Series Test

Directions: Show all work. Score___________.

1. If the first five terms of the Taylor series for $g(x)$ about $x=3$ are $3-7(x-3)-2(x-3)^2+\frac{5}{2}(x-3)^3+\frac{3}{4}(x-3)^4-6(x-3)^5$, then $g'''(3)=$

a) $\frac{1}{8}$ b) $\frac{3}{4}$ c) $\frac{9}{2}$ d) 6 e) 15

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2. For $-1\le x\le 1$, if $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{3n-2}}{3n-2}$, then $f'(x)=$

a) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{3n}$ b) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{3n-3}$ c) $\sum_{n=1}^{\infty}(-1)^{3n}x^{3n}$

d) $\sum_{n=1}^{\infty}(-1)^{n}x^{3n}$ e) $\sum_{n=1}^{\infty}(-1)^{n}x^{3n}$

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3. Give the first 3 nonzero terms in the Taylor series expansion about $x = 0$ for the function

$$f(x)=\cos(4x)$$

a) $1-8x^2+\frac{32}{3}x^4$ b) $1-8x^2+64x^4$ c) $x-\frac{32}{3}x^3+\frac{128}{15}x^5$

d) $1+4x+8x^2$ e) $1-8x^2$

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4. The Maclaurin expansion to approximate the area under the curve $e^{x^2}$ from $x=0$ to $x=1$ is

a) $1+\frac{1}{3}+\frac{1}{10}+\frac{1}{42}$ b) $1+1+\frac{1}{4}+\frac{1}{36}$ c) $1+1+\frac{1}{2}+\frac{1}{6}$

d) $1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}$ e) $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$

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5. A function $f(x)$ has the Maclaurin polynomial given by $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+...+\frac{x^{2n}}{(2n)!}+....$ Which of the following is an expression for $f(x)$?

a) $\cos x$ b) $e^x - \sin x$ c) $e^x + \sin x$ d) $\frac{1}{2}\left(e^x + e^{-x}\right)$ e) $e^{x^2}$

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6. If $g(x)=x\sin x$, which of the following is the power series expansion of $g'(x)$?

a) $1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+...$ b) $2x-\frac{4}{3!}x^3+\frac{6}{5!}x^5-\frac{8}{7!}x^7+...$

c) $x^2+\frac{1}{3!}x^4+\frac{1}{5!}x^6+\frac{1}{7!}x^8+...$ d) $1+2x+\frac{3}{2!}x^2+\frac{4}{3!}x^3+...$

e) $x^2-\frac{1}{3!}x^4+\frac{1}{5!}x^6-\frac{1}{7!}x^8+...$

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7. What are the values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{\sqrt[3]{n^4}}$ converges?

a) $4 < x < 6$ b) $4 \le x < 6$ c) $4 \le x \le 6$

d) $-1 \le x < 1$ e) $-1 \le x \le 1$

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8. The sum of the infinite series $\sum_{n=3}^{\infty} \frac{3(-2)^{n+1}}{5^n}$ is

a) 5 b) $\frac{15}{7}$ c) $\frac{16}{75}$ d) $\frac{48}{175}$ e) *dne*

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9. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^4-1}}$ II. $\sum_{n=3}^{\infty} \frac{1}{1+n^2}$ III. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)$

a) I only b) II only c) I and II only

d) I and III only e) I, II, and III

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10. The function $g(x)$ has a Taylor series about $x=-2$ that converges for all $x$ in the interval of convergence. Let $g(-2)=1$, $g'(-2)=-\frac{3}{2}$, and $g^{n}(-2)=(-1)^{n}\frac{(n-2)!}{5^{n}}$ for all $n\geq 2$.

a) Find the first four non-zero terms and the general term of the Taylor series for $g$ about $x=-2$

b) Find the interval of convergence of $g(x)$ about $x=-2$. Show the work that leads to your answer.

c) Use the first three non-zero terms of the Taylor polynomial for g to approximate $g(-1.9)$.

d) Determine is the approximation in part c) is within 0.001 of the actual value of $g(-1.9)$.

11. The function $g$ is continuous for all real numbers $x$ and is defined by

$$g(x)=\frac{e^{x^2}-1}{x^2} \text{ for } x\neq 0.$$

a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

b) Let $f$ be the function given by $f(x)=e^{x^2}$. Write the first four nonzero terms and the general term of the Taylor series for $f$ about $x=0$.

c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for $g$ about $x=0$.

d) Determine whether $g$ has a relative minimum, a relative maximum, or neither at $x=0$. Justify your answer.