AP Calculus BC '18-19 Name Solution Key

Taylor Series Test

Directions: Show all work. Score________.

1. If the first five terms of the Taylor series for $g(x)$ about $x=3$ are $3-7(x-3)-2(x-3)^2+\frac{5}{2}(x-3)^3+\frac{3}{4}(x-3)^4-6(x-3)^5$, then $g'''(3)=$

$\frac{g'''(3)}{3!} = \frac{5}{2}$

a) $\frac{1}{8}$ b) $\frac{3}{4}$ c) $\frac{9}{2}$ d) 6 e) 15

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2. For $-1 \le x \le 1$, if $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{3n-2}}{3n-2}$, then $f'(x)=$ $\frac{(3n-2)(-1)^{n+1}x^{3n-3}}{3n-2}$

a) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{3n}$ b) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{3n-3}$ c) $\sum_{n=1}^{\infty}(-1)^{3n}x^{3n}$

d) $\sum_{n=1}^{\infty}(-1)^{n}x^{3n}$ e) $\sum_{n=1}^{\infty}(-1)^{n}x^{3n}$

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3. Give the first 3 nonzero terms in the Taylor series expansion about $x = 0$ for the function

$$f(x)=\cos(4x) = 1-\frac{u^2}{2!}+\frac{u^4}{4!}-\frac{u^6}{6!}$$

a) $1-8x^2+\frac{32}{3}x^4$ b) $1-8x^2+64x^4$ c) $x-\frac{32}{3}x^3+\frac{128}{15}x^5$

d) $1+4x+8x^2$ e) $1-8x^2$

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4. The Maclaurin expansion to approximate the area under the curve e^{x^2} from $x=0$ to $x=1$ is

$A = \int_0^1 1+x^2+\frac{1}{2}x^4+\frac{1}{3!}x^6+\ldots$

a) $1+\frac{1}{3}+\frac{1}{10}+\frac{1}{42}$ b) $1+1+\frac{1}{4}+\frac{1}{36}$ c) $1+1+\frac{1}{2}+\frac{1}{6}$

d) $1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}$ e) $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$

$\int 1+(x^2)+\frac{(x^2)^2}{2!}+\frac{(x^2)^3}{3!}\,dx$

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5. A function $f(x)$ has the Maclaurin polynomial given by $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots+\frac{x^{2n}}{(2n)!}+\ldots$. Which of the following is an expression for $f(x)$?

a) $\cos x$ b) $e^x-\sin x$ c) $e^x+\sin x$ d) $\frac{1}{2}\left(e^x+e^{-x}\right)$ e) e^{x^2}

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6. If $g(x)=x\sin x$, which of the following is the power series expansion of $g'(x)$?

$g(x) = x\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\right) = x^2-\frac{x^4}{3!}+\frac{x^6}{5!}$

a) $1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots$ b) $2x-\frac{4}{3!}x^3+\frac{6}{5!}x^5-\frac{8}{7!}x^7+\ldots$ $g' = 2x - \ldots$

c) $x^2+\frac{1}{3!}x^4+\frac{1}{5!}x^6+\frac{1}{7!}x^8+\ldots$ d) $1+2x+\frac{3}{2!}x^2+\frac{4}{3!}x^3+\ldots$

e) $x^2-\frac{1}{3!}x^4+\frac{1}{5!}x^6-\frac{1}{7!}x^8+\ldots$

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7. What are the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{\sqrt[3]{n^4}}$ converges?

$\frac{1}{n^{4/3}}$ CONVERGES ABSOLUTELY

a) $4 < x < 6$ b) $4 \le x < 6$ c) $4 \le x \le 6$

d) $-1 \le x < 1$ e) $-1 \le x \le 1$

5 NOT AT CENTER

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8. The sum of the infinite series $\sum_{n=3}^{\infty} \frac{3(-2)^{n+1}}{5^n}$ is

$a_3 = \frac{3(-2)^4}{5^4}$

$r = -2/5$

a) 5 b) $\frac{15}{7}$ c) $\frac{16}{75}$ d) $\frac{48}{175}$ e) *dne*

$S = \frac{\frac{3(-2)^4}{5!}}{1-(-2/5)}$

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9. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^4 - 1}}$ II. $\sum_{n=3}^{\infty} \frac{1}{1+n^2}$ CONV III. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)$ DIV

DIVERGENCE TEST

a) I only b) II only c) I and II only

d) I and III only e) I, II, and III

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10. The function $g(x)$ has a Taylor series about $x=-2$ that converges for all x in the interval of convergence. Let $g(-2)=1$, $g'(-2)=-\frac{3}{2}$, and $g^n(-2)=(-1)^n\frac{(n-2)!}{5^n}$ for all $n\geq 2$.

$$C_n=\frac{(-1)^n(n-2)!}{5^n\, n!}=\frac{(-1)^n}{5^n(n)(n-1)}$$

a) Find the first four non-zero terms and the general term of the Taylor series for g about $x=-2$

$$1-\frac{3}{2}(x+2)+\frac{1}{50}(x+2)^2-\frac{1}{750}(x+2)^3+\cdots$$

$$\frac{(-1)^n}{5^n(n)(n-1)}(x+2)^n$$

b) Find the interval of convergence of $g(x)$ about $x=-2$. Show the work that leads to your answer.

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}}{5^{n+1}(n+1)(n)}(x+2)^{n+1}}{\frac{(-1)^n}{5^n n\cdot(n-1)}(x+2)^n}\right|=\lim_{n\to\infty}\frac{n(n-1)}{5(n+1)(n)}|x+2|<1$$

$$\frac{1}{5}|x+2|<1$$

$$|x+2|<5$$

$$-5<x+2<5$$

$$-7<x<3$$

$x=-7\rightarrow\frac{1}{n^2-n}$ CONV By LCT to $\frac{1}{n^2}$

$x=3\rightarrow\frac{(-1)^n}{n^2-n}$ CONV By AST

$$-7\leq x\leq 3$$

c) Use the first three non-zero terms of the Taylor polynomial for g to approximate $g(-1.9)$.

$\approx 1 - \frac{3}{2}(.1) + \frac{1}{50}(+.1)$

$= 1 - .15 + .0025$

$= .852$

d) Determine is the approximation in part c) is within 0.001 of the actual value of $g(-1.9)$.

$\left| \frac{1}{750}(+.1)^4 \right| < .001$

YES By ~~AST~~

11. The function g is continuous for all real numbers x and is defined by

$$g(x)=\frac{e^{x^2}-1}{x^2} \text{ for } x\neq 0.$$

a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{e^{x^2}\cancel{(2x)}}{\cancel{2x}} = \lim_{x\to 0} e^{x^2} = e^0 = 1$$

b) Let f be the function given by $f(x)=e^{x^2}$. Write the first four nonzero terms and the general term of the Taylor series for f about $x=0$.

$$e^u = 1+u+\frac{u^2}{2!}+\frac{u^3}{3!}+\cdots \frac{u^n}{n!}+\cdots$$

$$e^{x^2} = 1+x^2+\frac{x^4}{2}+\frac{x^6}{6}+\cdots \frac{x^{2n}}{n!}$$

c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

$$e^{x^2} - 1 = x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + \cdots + \frac{1}{n!}x^{2n} + \cdots$$

$$g(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^4 + \frac{1}{n!}x^{2n-2}$$

d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

$g'(0) = 0$ BECAUSE THERE IS NO x^1 TERM

$\frac{g''(0)}{2!} = \frac{1}{2} > 0 \therefore g(0)$ IS A RELATIVE MINIMUM BY THE 2^{ND} D_x TEST