BC Calculus '19-20 name____________________________
Advanced Integration Techniques Test

Score___________

1. $\int x^2 \ln x \, dx =$

(a) $\frac{1}{9}x^3(3\ln x - 1) + c$ (b) $\frac{1}{9}x^3(\ln x - 3) + c$

(c) $\frac{1}{2}x^2(2\ln x - 1) + c$ (d) $\frac{1}{2}x^2(2\ln x + 1) + c$ (e) $\frac{1}{2}(\ln x)^2 + c$

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2. $\int \frac{6}{x^2 + 2x - 8} dx =$

(a) $\ln\left|(x-2)(x+4)\right| + c$ (b) $6\ln\left|(x-2)(x+4)\right| + c$

(c) $\ln\left|\frac{x-2}{x+4}\right| + c$ (d) $\ln\left|\frac{x+4}{x-2}\right| + c$ (e) $6\ln\left|\frac{x-2}{x+4}\right| + c$

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3. $\int \frac{t}{t+5}\,dt =$

(a) $\ln|t+5|+c$ (b) $\frac{t^2}{2}\ln|t+5|+c$ (c) $t-5\ln|t+5|+c$

(d) $-5\ln|t+5|+c$ (e) $t+5\ln|t+5|+c$

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4. The growth rate of a population $y(t)$ of dolphins is modeled by the logistic growth equation $\frac{dy}{dt}=\frac{y}{2}(120-y)$. If $y(0)=30$, which of these describes the future behavior of the population?

a) The population will increase towards 60 dolphins
b) The population will increase towards 120 dolphins
c) The population will decrease towards 120 dolphins
d) The population will decrease towards 60 dolphins
e) None of these

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5. $\int \frac{e^x}{e^{2x}+49}\,dx =$

(a) $\frac{1}{7}\tan^{-1}\left(\frac{e^x}{7}\right)+c$ (b) $\frac{1}{7}\tan^{-1}\left(\frac{\sqrt{e^x}}{7}\right)+c$

(c) $\frac{1}{14}\ln\left|\frac{e^x-7}{e^x+7}\right|+c$ (d) $\frac{1}{2}\ln\left|e^{2x}+49\right|+c$ (e) $\frac{1}{14}\tan^{-1}\left(\frac{e^x}{7}\right)+c$

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6. What is the solution curve to the logistic growth equation $\frac{dy}{dt} = 10y\left(1 - \frac{y}{100}\right)$ given that $y(0) = 20$?

(a) $y = \frac{100}{1 + 4e^{-0.1t}}$ (b) $y = \frac{100}{1 + 4e^{10t}}$ (c) $y = \frac{100}{1 + 4e^{-10t}}$

(d) $y = \frac{100}{1 + 20e^{-10t}}$ (e) $y = \frac{100}{1 + 20e^{-0.1t}}$

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7. $\int \frac{6}{(2k-1)^2 - 9}\, dk =$

(a) $\frac{1}{2}\ln\left|\frac{2k-4}{2k+2}\right| + c$

(b) $\ln\left|\frac{2k-4}{2k+2}\right| + c$

(c) $\ln\left|\frac{2k-10}{2k+8}\right| + c$

(d) $\tan^{-1}\left(\frac{2k-1}{3}\right) + c$

(e) $2\tan^{-1}\left(\frac{2k-1}{3}\right) + c$

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8. Which of the following statements are **false**?

I. $\int \frac{1}{x\sqrt{16-x^2}}\,dx = \sec^{-1}\frac{x}{4} + c$

II. $\int \cot x\,dx = \ln|\sin x| + c$

III. $\int \left(\frac{2e^x+1}{e^x}\right) dx = 2x + \ln|e^x| + c$

a) I only b) II only c) III only

d) I and II only e) I and III only f) I, II, and III

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9. What is the best method to integrate $\int \frac{5}{x^2+x+5}\,dx$?

(a) Integration by Parts

(b) Partial Fractions

(c) U-Substitution

(d) Complete the Square

(e) Long Division

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1. Find the volume of the solid formed by revolving the region bounded by the x-axis, $x = 0$, $x = \frac{1}{3}$, and $y = \sqrt{\cos^{-1}(3x)}$ around the x-axis.

2. $\int \cos\left(6\theta\right) e^{2\theta}\, d\theta$

3. $\displaystyle\int \frac{x+14}{2x^2-7x-4}\,dx$

4. Like stars, black holes accrete (gain) mass from solar gasses. It was always assumed that this accretion was never-ending, ultimately, black holes would swallow the universe. But the research into supermassive black holes (SMBH) by Columbia Inayoshi and Haiman seems to indicate that there is a limit to how large these black holes can get. SMBH sizes are on the scale of $10^9 M_{suns}$ (solar masses). For simplicity, we will refer to these units as Kellar-masses. The research seems to indicate that the size limit for SMBHs is 1000 Kellar-masses. If the growth were logistic, one model might be

$$\frac{dM}{dt} = .256M(1000 - M).$$

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a) If $M(0) = 10$, how many Kellar-masses would a SMBH attain when the accretion rate was the highest?

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b) If $M(0) = 10$, state the particular solution to the logistic differential equation.

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c) Further research might show the growth to be exponential rather than logistic. Assuming a new model of $\frac{dM}{dt} = .256(1100 - M)$, what would be $\lim_{t \to \infty} M$?

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d) If $M(0)=10$ and $\frac{dM}{dt}=.256(1100-M)$, state the particular solution to the logistic differential equation.