AP Calculus BC '19-20
Anti-Derivative Test

Name Solution Key

Score___________

1. $\int \frac{\ln(3x)}{x} dx =$ $\int u\,du = \frac{u^2}{2} + c$ $u = \ln(3x)$
$du = \frac{1}{3x}(3)\,dx = \frac{1}{x}\,dx$

(A) $\frac{\ln^2(3x)}{2} + c$ (B) $\frac{\ln^2(3x)}{x^2} + c$ (C) $\frac{\ln^2(3x)}{6} + c$

(D) $\frac{3\ln^2(3x)}{2} + c$ (E) $\frac{1-\ln(3x)}{x^2} + c$

---

2. $\int \sin^2 x\, dx =$

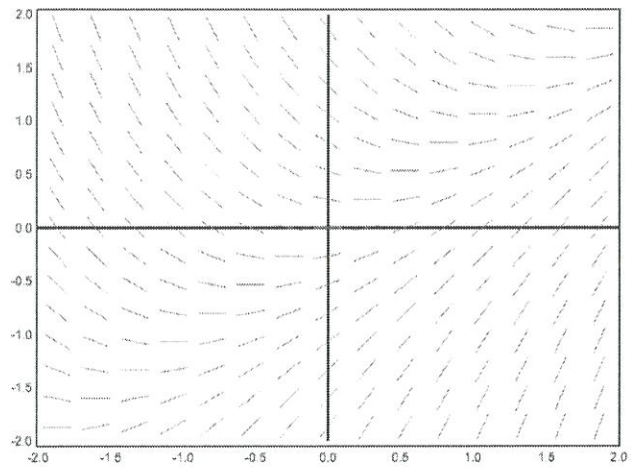
(A) $\frac{1}{2}(1+\cos 2x)+c$ (B) $\frac{1}{2}(x+\sin 2x)+c$ (C) $\frac{1}{2}(1-\cos 2x)+c$

(D) $\frac{1}{2}(x+\frac{1}{2}\sin 2x)+c$ (E) $\frac{1}{2}(x-\frac{1}{2}\sin 2x)+c$

$\int \sin^2 u\, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + c = \frac{1}{2}\left(u - \frac{1}{2}\sin 2u\right) + c$

---

3. Which of the following differential equations corresponds to this slope field?



$m = 0$ WHEN $x = y$

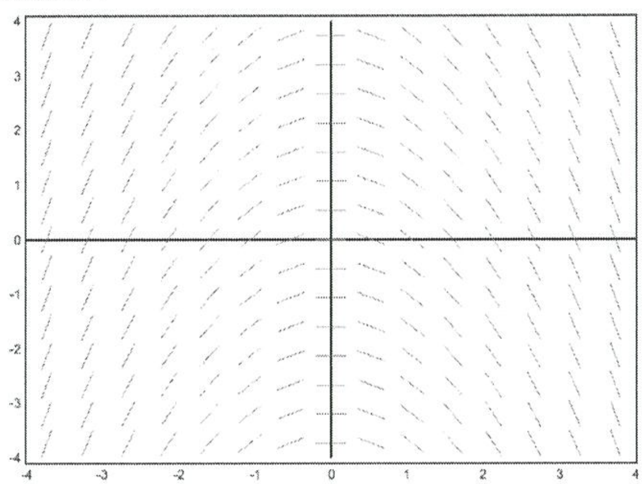
$y > x \Rightarrow$ NEG m

(A) $\dfrac{dy}{dx} = x$ (B) $\dfrac{dy}{dx} = xy$ (C) $\dfrac{dy}{dx} = y - x$

(D) $\dfrac{dy}{dx} = x + y$ (E) $\dfrac{dy}{dx} = x - y$

---

4. Which of the following equations might be the solution to the slope field shown in the figure below?



(A) $y = -x$ (B) $y = -x^2$ (C) $y = e^x$ (D) $y = x^3$ (E) $y = \sqrt{x}$

---

5. The solution curve to the differential equation $\frac{dy}{dx} = y$ through the point $(-2,-1)$ is:

(A) $y = -e^{x+2}$ (B) $y = e^{x+2}$ (C) $y = -e^{x} + e^{-2}$

(D) $y = \sqrt{\frac{-1}{x+1}}$ (E) $y = -\sqrt{\frac{-1}{x+1}}$

$\frac{1}{y}\,dy = dx$

$\ln|y| = x + c \qquad |y| = ke^{x}$

$(-2,-1) \Rightarrow -1 = ke^{-2} \quad -k = e^{2}$

---

6. $\int \sqrt[4]{5^x}\,dx = \int 5^{x/4}\,dx = 4\int 5^{u}\,du = 4 \cdot \frac{5^{u}}{\ln 5}$

a) $4\sqrt[4]{5^x} + C$ b) $\frac{4}{\ln 5}\sqrt[4]{5^x} + C$ c) $\frac{\sqrt[4]{5^x}}{\ln 5} + C$

d) $\sqrt[4]{5^x} + C$ e.) $\frac{\sqrt[4]{5^x}}{4\ln 5} + C$

7. $\frac{1}{5}\int \csc(5\theta)\,5\,d\theta =$ $= \frac{1}{5}\ln|\csc 5\theta - \cot 5\theta| + c$

a) $-\csc(5\theta)\cot(5\theta)+c$

b) $\frac{1}{10}\csc^2(5\theta)+c$

c) $\ln|\csc(5\theta)-\cot(5\theta)|+c$

d) $-\frac{1}{5}\csc(5\theta)\cot(5\theta)+c$

e) $\frac{1}{5}\ln|\csc(5\theta)-\cot(5\theta)|+c$

---

8. Identify the **first** mistake (if any) in this process:

$$\frac{dy}{dx} = 2x\cos^2 y$$

✓ Step 1: $(\sec^2 y)dy = 2xdx$
✓ Step 2: $\int(\sec^2 y)dy = \int 2x\,dx$
~~Step 3:~~ $\cancel{\tan^2} y = x^2 + c$
Step 4: $y = \cancel{\tan^{-2}}(x^2+c)$

a) Step 1

b) Step 2

c) Step 3

d) Step 4

e) No Mistake

---

9. $\displaystyle\int\left(\frac{1}{(1-x)^2}+\frac{x}{1+x^2}+\frac{1}{1+x^2}\right)dx$

$$= -\int \frac{1}{(1-x)^2}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

$$= -\int u^{-2} + \frac{1}{2}\int \frac{1}{u}\,du$$

$$= -\frac{u^{-1}}{-1} + \frac{1}{2}\ln|u| + \tan^{-1}x + C$$

$$= \frac{1}{1-x} + \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C$$

---

10. $\displaystyle\int \frac{-y^5\,dy}{\sqrt{y^3+5}}$

$u = y^3+5 \rightarrow y^3 = u-5$

$du = 3y^2\,dy$

$$= -\frac{1}{3}\int \frac{y^3\,3y^2\,dy}{(y^3+5)^{1/2}} = \frac{-1}{3}\int \frac{(u-5)}{u^{1/2}}\,du$$

$$= \frac{-1}{3}\int \left(u^{1/2} - 5u^{-1/2}\right) du$$

$$= \frac{-1}{3}\left(\frac{u^{3/2}}{3/2} - \frac{5u^{1/2}}{1/2}\right) + C$$

$$= \frac{-2}{9}(y^3+5)^{3/2} + \frac{10}{3}(y^3+5)^{1/2} + C$$

11. A particle's acceleration is given by $a(t) = 3t^2 + 1$ meters per second squared. At time $t = 1$, the particle's velocity is 4 meters per second and its position is 0 meters. Find the particle's position at time $t = 2$.

$V = \int (3t^2 + 1)\,dt = t^3 + t + c_1$

$v(t) = t^3 + t + 2$

$t=1,\ v=4 \Rightarrow 4 = 1 + 1 + c_1 \Rightarrow c_1 = 2$

$x(t) = \int (t^3 + t + 2)\,dt$

$= \frac{1}{4}t^4 + \frac{1}{2}t^2 + 2t + c_2$

$t = 1,\ x = 0 \Rightarrow 0 = \frac{1}{4} + \frac{1}{2} + 2 + c_2 \Rightarrow c_2 = -\frac{11}{4}$

$x(t) = \frac{1}{4}t^4 + \frac{1}{2}t^2 + 2t - \frac{11}{4}$

$x(2) = 4 + 2 + 4 - \frac{11}{4} = \frac{29}{4}$

---

12. $\int \sec^6(\theta)\tan^4(\theta)\,d\theta$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\sec^2\theta = \tan^2\theta + 1 = u^2 + 1$

$= \int \sec^4\theta \tan^4\theta \sec^2\theta\,d\theta$

$= \int (1 + u^2)^2 u^4\,du$

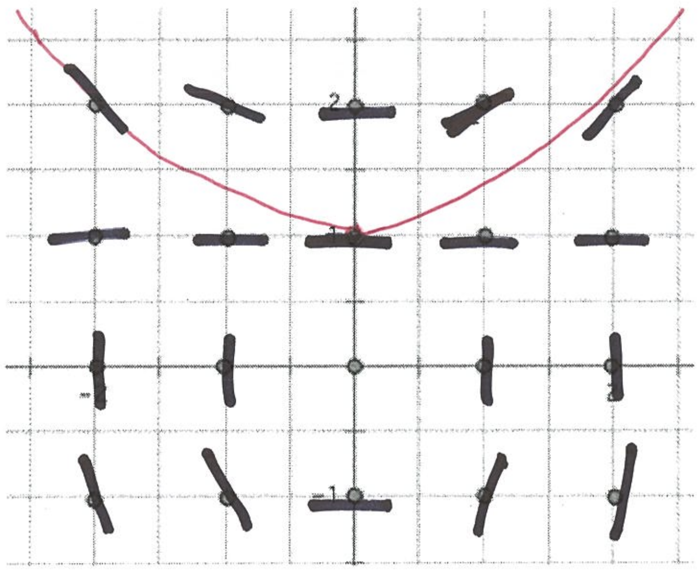
$= \int (1 + 2u^2 + u^4)(u^4)\,du$

$= \int (u^4 + 2u^6 + u^8)\,du$

$= \frac{1}{5}u^5 + \frac{2}{7}u^7 + \frac{1}{9}u^9 + c = \frac{1}{5}\tan^5\theta + \frac{2}{7}\tan^7\theta + \frac{1}{9}\tan^9\theta + c$

13. Given the differential equation, $\frac{dy}{dx} = \frac{x(y-1)}{y}$

a. On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



b. If the solution curve passes through the point (0, 1), sketch the solution curve on the same set of axes as your slope field.

14. Find the particular solution $w = f(t)$ that passes through $\left(0, \frac{\sqrt{3}}{4}\right)$ if

$$\frac{dw}{dt} = t^3\sqrt{1-4w^2}$$

$$\frac{1}{2}\int \frac{2 \cdot 1}{\sqrt{1-4w^2}}\,dw = \int t^3\,dt$$

$$\frac{1}{2}\sin^{-1}(2w) = \frac{t^4}{4} + C$$

$$\left(0, \sqrt{3}/4\right) \to \frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2} = 0 + C$$

$$\frac{\pi}{6} = \frac{1}{2}\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2}\sin^{-1}2w = \frac{t^4}{4} + \frac{\pi}{6}$$

$$\sin^{-1}2w = \frac{1}{2}t^2 + \frac{\pi}{3}$$

$$2w = \sin\left(\frac{1}{2}t^2 + \frac{\pi}{3}\right)$$

$$w = \frac{1}{2}\sin\left(\frac{1}{2}t^2 + \frac{\pi}{3}\right)$$