

AP Calculus BC '19-20
Integral Test

Name _____

Score _____

NO CALCULATOR ALLOWED

1. Find $\int_1^e \frac{1}{x} dx$

- a. 1 b. e c. $\frac{1}{2}$ d. -1 e. -2
-

2. $\int_0^{\frac{\pi}{3}} \sec x \tan x (1 + \sec x) dx$

- a. 4 b. $\frac{5}{2}$ c. 3 d. $\frac{9}{2}$ e. 5
-

3. $\int_0^1 \frac{2}{1+x^2} dx$

- a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{6}$ d. 1 e. 2
-

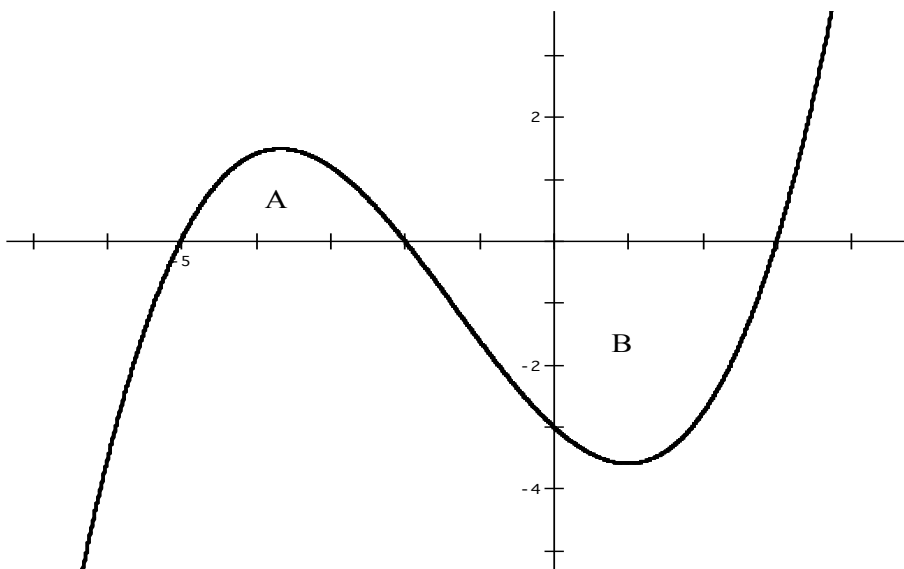
4. The average value of $y = \cos x (\sin(\sin x))$ on $x \in \left[0, \frac{\pi}{2}\right]$ is

- a. 1 b. $\frac{2}{\pi} - \frac{1}{2}$ c. $\frac{2}{\pi} \cos 1$ d. $\frac{2}{\pi} - \frac{2}{\pi} \cos 1$ e. $\frac{1}{2} - \frac{2}{\pi} \cos 1$
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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x) dx$?

- a) The change in temperature during the first t hours.
- b) The change in temperature during the first day.
- c) The average rate at which the temperature changed during the first t hours.
- d) The rate at which the temperature is changing during the first day.
- e) The rate at which the temperature is changing at the end of the 24th day.

6. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x -axis.



In terms of A and B, $2\int_{-5}^3 f(x) dx - \int_{-2}^3 f(x) dx =$

- a. A
- b. $A - B$
- c. $2A - B$
- d. $A + B$
- e. $A + 2B$

7. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoidal Sum is used to approximate $\int_1^5 f(x) dx$, the result is

- a) 3.7 b) 4.5 c) 4.6 d) 5.2
e) none of these
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CALCULATOR ALLOWED

Directions: Show all work.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

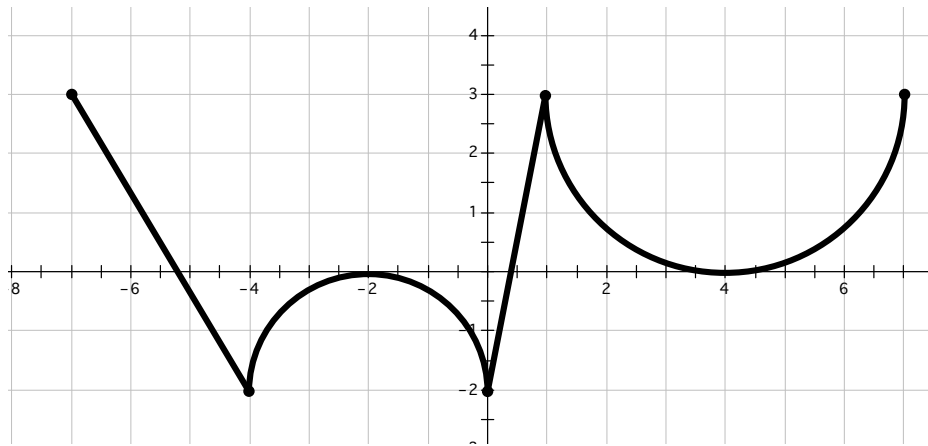
a. Use the data in the table to estimate $W'(t)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

b. Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.

c. For $0 \leq t \leq 20$, the average temperature of the water in the tub is

$\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

d. For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?



a) Find $g(0)$, $g'(0)$, and $g''(0)$.

b) Find the equation of the line tangent to $g(x)$ at $x = 0$.

c) At what x -values is $g(x)$ decreasing and concave up? Justify your answer.

d) Find the x -coordinate of the absolute maximum of $g(x)$. Justify your answer.

6. A tank at a sewage processing plant contains 125 gallons of raw sewage at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, sewage is pumped into the tank at the rate $E(t) = 2 + \frac{10}{1 + \ln(t+1)}$. During the same time interval, sewage is pumped out at a rate of $L(t) = 12 \sin\left(\frac{t^2}{47}\right)$.

a) How many gallons of sewage are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?

b) Is the level of sewage rising or falling at $t = 6$? Explain your reasoning.

c) How many gallons of sewage are in the tank at $t = 12$ hours?

d) At what time t , for $0 \leq t \leq 12$, is the volume of the sewage at an absolute maximum? Show the analysis that leads to your answer. If the sewage level ever exceeds 150 gallons, the tank overflows. Is there a time at which the tank overflows? Explain.
