AP Calculus BC '19-20
Integral Test

Name Solution Key

Score______________

NO CALCULATOR ALLOWED

1. Find $\int_1^e \frac{1}{x}\,dx$ $= \ln x \Big]_1^e = \ln e - \ln(1) = 1$

a. 1 b. e c. $\frac{1}{2}$ d. -1 e. -2

$u = 1+\sec x$

2. $\int_0^{\frac{\pi}{3}} \sec x \tan x(1+\sec x)\,dx$ $= \int_2^3 u\,du = \frac{u^2}{2}\Big]_2^3$

a. 4 b. $\frac{5}{2}$ c. 3 d. $\frac{9}{2}$ e. 5

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3. $\int_0^1 \frac{2}{1+x^2}\,dx$ $= 2\tan^{-1} x\Big]_0^1 = 2(\tan^{-1} 1 - \tan^{-1} 0) = 2\left(\frac{\pi}{4}\right)$

a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{6}$ d. 1 e. 2

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4. The average value of $y = \cos x(\sin(\sin x))$ on $x \in \left[0, \frac{\pi}{2}\right]$ is

a. 1 b. $\frac{2}{\pi} - \frac{1}{2}$ c. $\frac{2}{\pi}\cos 1$ d. $\frac{2}{\pi} - \frac{2}{\pi}\cos 1$ e. $\frac{1}{2} - \frac{2}{\pi}\cos 1$

$$\frac{1}{\pi/2 - 0}\int_0^{\pi/2} \cos x\,(\sin(\sin x))\,dx = \frac{2}{\pi}\int_0^1 \sin u\,du$$

$$= \frac{2}{\pi}\Big[-\cos u\Big]_0^1$$

5. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)\,dx$?

a) The change in temperature during the first t hours.
b) The change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24th day.

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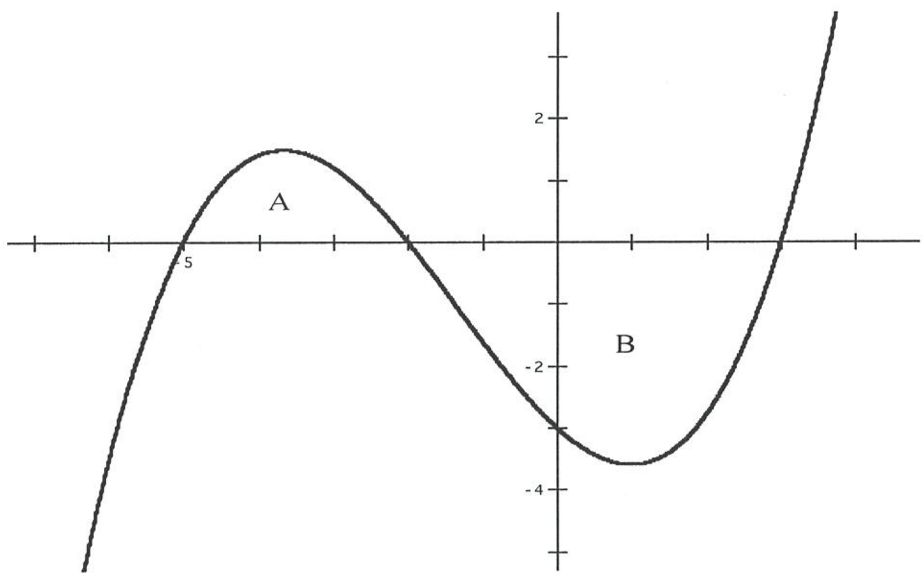
6. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x-axis.



In terms of A and B, $2\int_{-5}^{3} f(x)\,dx - \int_{-2}^{3} f(x)\,dx =$ $2(A-B)-(-B) = 2A \mp B$

a. A b. $A-B$ c. $2A-B$ d. $A+B$ e. $A+2B$

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7. The following table lists the known values of a function $f(x)$.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoidal Sum is used to approximate $\int_1^5 f(x)\,dx$, the result is

a) 3.7 b) 4.5 c) 4.6 d) 5.2

e) none of these

$\cancel{\frac{1}{2}(1+2)(1)+\frac{1}{2}(2+3)(1)+\frac{1}{2}(3+4)(1)}$

$\frac{1}{2}(0+1.1)(1)+\frac{1}{2}(1.1+1.4)(1)+\frac{1}{2}(1.4+1.2)(1)+$

$\frac{1}{2}(1.4+1.2)(1)+\frac{1}{2}(1.2+1.5)(1)$

$=4.45$

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CALCULATOR ALLOWED
Directions: Show all work.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55F°. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

2

a. Use the data in the table to estimate $W'(t)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) = \frac{W(15)-W(9)}{15-9} = 1.017$$

THE TEMPERATURE OF THE WATER IS INCREASING AT 1.017 °F/MIN AT t=12 MIN

2

b. Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(20) - W(0) = 16\ ^\circ F$$

THE WATERS TEMPERATURE ~~RAISE~~ INCREASED BY 16°F OVER THE FIRST 20 MINUTES

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c. For $0 \leq t \leq 20$, the average temperature of the water in the tub is

3 $\frac{1}{20}\int_0^{20} W(t)dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\int_0^{20} W(t)\,dt \approx 4(55)+5(57.1)+6(61.8)+5(67.9)=1215.8$$

$$\frac{1}{20}\int_0^{20} W(t)\,dt \approx \frac{1}{20}(1215.8)=60.79$$

UNDERESTIMATE BECAUSE THE FUNCTION IS INCREASING.

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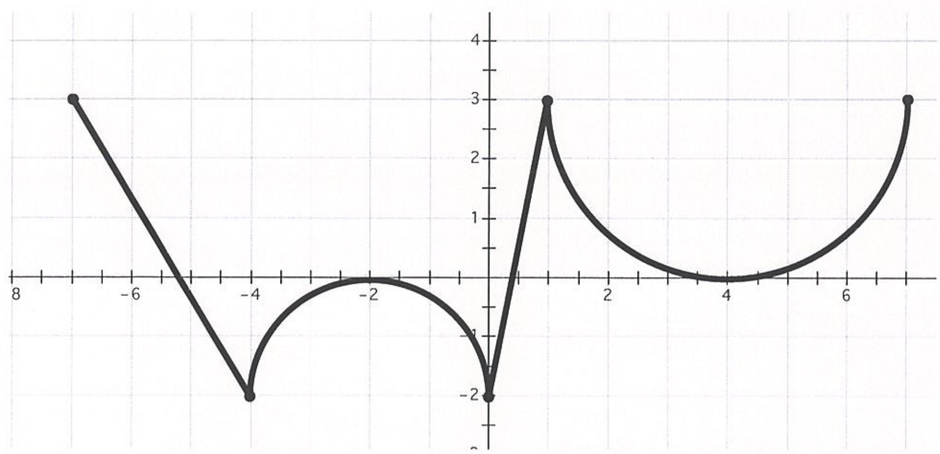
d. For $20 \leq t \leq 25$, the function W that models the water temperature has first

2 derivative given by $W'(t)=0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t=25$?

$$W(25)=71+\int_{20}^{25} W'(t)\,dt$$

$$=73.043$$

Let $g(x)=\int_{-4}^{x} f(t)\,dt$ for $-7 \le t \le 7$, where the graph of the differentiable function f is shown below.



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a. Find $g(0)$, $g'(0)$, and $g''(0)$.

$2\pi - 8$ / -2 / DNE

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b. Find the equation of the line tangent to $g(x)$ at $x=0$.

$y - (2\pi - 8) = -2(x - 0)$

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c. At what x-values is $g(x)$ decreasing and concave up? Justify your answer.

$x \in (-4, -2), (0, .4)$

$f = g'$; f is NEG & INCREASING

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d. Find the x-coordinate of the absolute maximum of $g(x)$. Justify your answer.

x	g
-5.25	1
7	6.705

$x = 7$

6. A tank at a sewage processing plant contains 125 gallons of raw sewage at time $t = 0$. During the time interval $0 \le t \le 12$ hours, sewage is pumped into the tank at the rate $E(t) = 2 + \frac{10}{1+\ln(t+1)}$. During the same time interval, sewage is pumped out at a rate of $L(t) = 12\sin\left(\frac{t^2}{47}\right)$.

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a) How many gallons of sewage are pumped into the tank during the time interval $0 \le t \le 12$ hours?

$$\int_0^{12} E(t)\,dt = 70.571$$

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b) Is the level of sewage rising or falling at $t = 6$? Explain your reasoning.

$\cancel{E(6) = 5.395}$ WHICH IS < 0 $\therefore$ LEVEL IS ~~RISING~~ FALLING

$E(6) - L(6) = -2.924$

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c) How many gallons of sewage are in the tank at $t = 12$ hours?

$$125 + \cancel{70.5871} + \int_0^{12} E(t) - L(t)\, dt$$

$\cancel{197.5744}$ ≈ 122.974

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d) At what time t, for $0 \le t \le 12$, is the volume of the sewage at an absolute maximum? Show the analysis that leads to your answer. If the sewage level ever exceeds 150 gallons, the tank overflows. Is there a time at which the tank overflows? Explain.

$E(t) - L(t) = 0 \Rightarrow t = 4.978723$ or $t = 11.31847$

t	TOTAL
0	125
4.978	149.366
11.318	120.738
12	122.974

ABS MAX = 149.366

NO, IT DOES NOT OVERFLOW
