AP Calculus BC '19-20 Name Solution Key
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Derivative Test

1.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	3	4	-1
2	4	-1	7	8
4	1	2	2	1

Selected values of f, g, and their derivatives are indicated in the table above. Let $h(x)=g\left(f\left(x^2\right)\right)$. What is the value of $h'(2)$?

a) -8 b) -4 c) 1 d) 2 e) 16

$g'(f(x^2))\cdot f'(x^2)\,(2x)$

$= g'(f(4))\cdot f'(4)\cdot 4 = g'(1)\cdot f'(4)\,(4)$

$=(-1)\,(2)\,(4) = -8$

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2. Differentiate $y=x^2\tan^{-1}\left(\frac{1}{x}\right)$

a) $\dfrac{2x}{1+x^2}$

b) $\dfrac{2\tan^{-1}\frac{1}{x}}{x}$

c) $\dfrac{x^2}{x^2+1}-2x\tan^{-1}\dfrac{1}{x}$

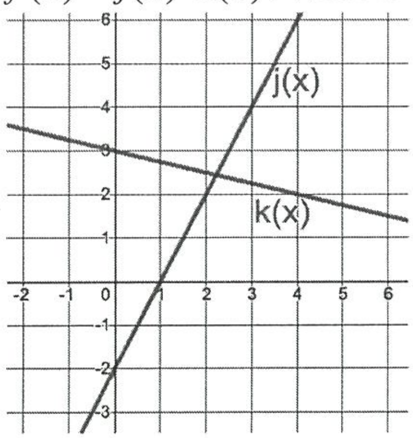
d) $\dfrac{-x^2}{x^2+1}+2x\tan^{-1}\dfrac{1}{x}$

e) $\dfrac{-1}{x^2+1}+2x\tan^{-1}\dfrac{1}{x}$

$\dfrac{dy}{dx} = x^2\,\dfrac{1}{1+\left(\frac{1}{x}\right)^2}\left(\dfrac{-1}{x^2}\right)+2x\tan^{-1}\dfrac{1}{x}$

$= \dfrac{-x^2}{x^2+1}+2x\tan^{-1}\left(\dfrac{1}{x}\right)$

3. The graphs of linear functions $j(x)$ and $k(x)$ are shown below. Let $f(x) = j(x) \cdot k(x)$. What is the equation of the line tangent to $f(x)$ at $x = 0$?



$f'(0) = j(0) \cdot k'(0) + k(0)j'(0)$

$= (-2)(-1/4) + 3(2) = \frac{13}{2}$

$f(0) = (-2)(2) = -4$

$y + 4 = \frac{13}{2}(x - 0)$

a) $y = \frac{11}{2}x - 6$ b) $y = -\frac{1}{2}x - 6$ c) $y = \frac{13}{2}x - 6$

d) $y = \frac{7}{4}x + 6$ e) $y = -\frac{2}{3}x + 6$

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4. The derivative of the function $g(x)$ is given by $\frac{dy}{dx} = xy$. Using Euler's Method with 3 steps of equal size, starting at the point $(2,1)$, $g(5) \approx$

a) 3 b) 6 c) 7 d) 60 e) 63

POINT	xy	TAN LINE	NEW x	NEW y
(2, 1)	2	$y - 1 = 2(x - 2)$	3	3
(3, 3)	9	$y - 3 = 9(x - 3)$	4	12
(4, 12)	48	$y - 12 = 48(x - 4)$	5	60

5. Find the slope of the line **normal** to the curve $\sqrt{x}-\cos y=x$ at the point $\left(1,\dfrac{\pi}{2}\right)$

a) -2 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) 2 e) DNE

$\frac{1}{2}x^{-1/2}+\sin y\frac{dy}{dx}=1$

$(1,\pi/2)\rightarrow \frac{1}{2}+1\frac{dy}{dx}=1$ $\frac{dy}{dx}=1/2=m_{TAN}$

m_{TAN} $m_{NORM}=-2$

6. Find the approximate value of $\sqrt[3]{7}$ using the tangent approximation for $\sqrt[3]{x}$ at $x=8$.

a) $\frac{25}{12}$ b) $\frac{23}{12}$ c) $\frac{15}{8}$ d) $\frac{7}{4}$ e) $\frac{13}{8}$

$x=8\rightarrow\sqrt[3]{8}=2=y$

$\frac{dy}{dx}=\frac{1}{3}x^{-2/3}\rightarrow m=\frac{1}{12}$

$y-2=\frac{1}{12}(x-8)$

$y(7)=2-\frac{1}{12}=\frac{23}{12}$

FREE RESPONSE – show all work in a clear, organized manner. Simplify answers.

7a. $\frac{d}{dx}\left(\cos^{-1}4x^2\right) = \frac{-1}{\sqrt{1-16x^4}}(8x) = \frac{-8x}{\sqrt{1-16x^4}}$

b. $\frac{d}{dx}\left(\ln\left(x^2+2x-8\right)\right) = \frac{2x+2}{x^2+2x-8}$

c. $\frac{d}{dx}\left(e^{-\frac{1}{2}x}\csc x\right) = e^{-\frac{1}{2}x}(-\csc x\cot x) + \csc x\left(-\frac{1}{2}e^{-1/2x}\right)$

$= -\csc x\, e^{-\frac{1}{2}x}\left[\cot x + \frac{1}{2}\right]$

d. $\frac{d}{dx}\left(\frac{-5x}{25+x^2}\right) = \frac{(25+x^2)(-5)-(-5x)(2x)}{(25+x^2)^2}$

$= \frac{5x^2-125}{(x^2+25)^2} = \frac{5(x^2-25)}{(x^2+25)^2}$

8. If $f(x) = \tan^2(3x)$, find $f'(x)$ and $f''(x)$. Write answers in simplified, factored form.

$f' = 2\tan 3x(\sec^2 3x)(3) = 6\tan 3x \sec^2 3x$

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$f'' = 6\tan 3x\,[2\sec 3x \sec 3x \tan 3x\,(3)] + \sec^2 3x\,(6\sec^2 3x\,(3))$

$= 36\tan^2 3x \sec^2 3x + 18\sec^4 3x$

$= \cancel{18\sec^2 3x\,[\tan^2 3x}$

$= 18\sec^2 3x\,[2\tan^2 3x + \sec^2 3x]$

9. Find the second derivative of $y = \ln(\pi \sin x)$. Simplify your answer.

$\frac{dy}{dx} = \frac{1}{\pi \sin x}\pi \cos x = \cancel{\frac{\pi}{\pi}} \cot x$

$\frac{d^2y}{dx^2} = \cancel{\frac{\pi}{\pi}}(-\csc^2 x) = \cancel{\frac{1}{\pi}}\csc^2 x$

10. Consider the function $\frac{d}{dx}\left(y^3 - 2xy = 1 - x^2.\right)$

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a) Show that $\dfrac{dy}{dx} = \dfrac{-2x+2y}{3y^2-2x}$

$$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} - 2y = -2x$$

$$(3y^2-2x)\frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} = \frac{-2x+2y}{3y^2-2x}$$

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b) Find the approximate value of $f(-1.2)$, using the linear approximation at $(-1,0)$

$$m(-1) = \left.\frac{dy}{dx}\right|_{x=-1} = 1$$

$$y - 0 = x + 1$$

$$y(-1.2) = -.2$$

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c) Find $\dfrac{d^2y}{dx^2}$ at $(-1,0)$.

$$\frac{d^2y}{dx^2} = \frac{(3y^2-2x)\left(-2+2\frac{dy}{dx}\right) - (-2x+2y)\left(6y\frac{dy}{dx}-2\right)}{(3y^2-2x)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-1,0)} = \frac{2(-2+2) - (2+0)(-2)}{4} = 1$$

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d) Is your approximation from part (b) higher or lower than the actual value of $f(-1.2)$? Justify your response.

$f'' > 0 \therefore$ ~~ESTIMA~~ THE TANGENT LINE IS UNDER THE CURVE.

$f(-1.2)$ IS AN UNDER APPROXIMATION.