BC Calculus '19-20
Dx Apps Test
No Calculator

Name Solution Key

1. On which of the following interval is the graph of $y = 2x^3 - 3x^2 - 12x + 15$ both decreasing AND concave up?

$y' = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

$y'' = 12x - 6$

$y' < 0$ $y'' > 0$

a) $(-\infty, -1)$ b) $\left(-1, -\frac{1}{2}\right)$ c) $(-1, 2)$

d) $\left(\frac{1}{2}, 2\right)$ (e) $(2, \infty)$

$y'$: + 0 − 0 + ; −1, 2

$y''$: − + ; $\frac{1}{2}$

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

| $x$ | $f'(x)$ | $f''(x)$ | $g'(x)$ | $g''(x)$ |
|---|---|---|---|---|
| 2 | 0 | -2 | 8 | 0 |
| 4 | 8 | 0 | 0 | 3 |
| 8 | 0 | -12 | 0 | 4 |

Then at $x = 4$, $f(x)$ has a:

a) Relative Maximum b) Relative Minimum

c) Point of Inflection d) Zero

e) None of these

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3. Suppose $f'(x) = \dfrac{(x+1)^3(x-4)^5}{(x^3+1)}$. Which of the following statements must be **true**?

$(x^2-x+1)(x+1)$   $f'$: $-$ 0 $-$ 0 $+$ at $-1$, $4$

~~I.~~ The slope of the line tangent to $y = f(x)$ at $x = -1$ is 36.

II. $f(x)$ is decreasing on $x \in (1, 4)$

~~III.~~ $f(x)$ has a maximum at $x = 4$

a) I only  b) II only  c) III only  d) I and II  e) II and III only

ab) I and III only  ac) I, II, and III  ad) None of these

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4. A particle moves along a straight line with velocity given by $v(t) = 5 + 4t - t^2$. When is the particle *slowing down*?

$-(t+1)(t-5)$   $v(t)$: $-$ 0 $+$ 0 $-$ at $-1$, $5$

$a(t) = 4 - 2t$   $+$ 0 $-$ at $2$

a) $t \in (-\infty, 1)$

b) $t \in (-\infty, -1) \cup (5, \infty)$

c) $t \in (-1, 2) \cup (5, \infty)$

d) $t \in (-\infty, -1) \cup (2, 5)$

e) $t \in (5, \infty)$

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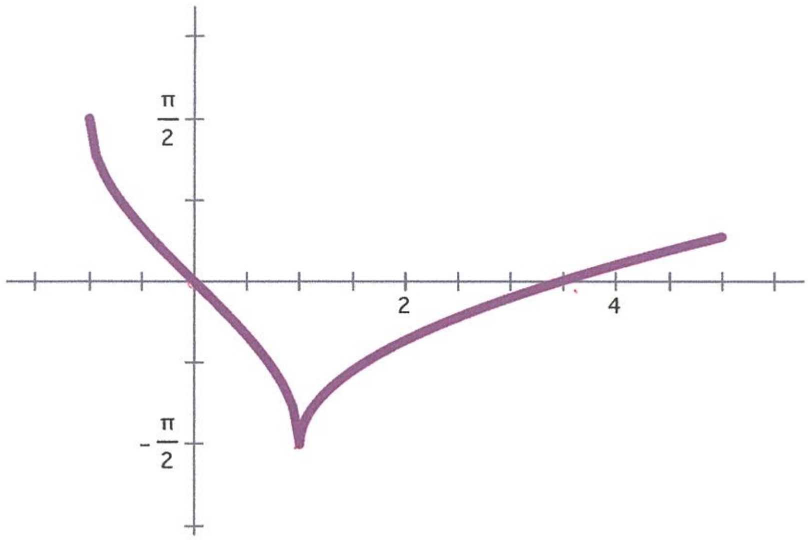
5. This is the graph of $g'(x)$, the derivative of $g(x)$.




Which of the following sign patterns are hidden with the graph.

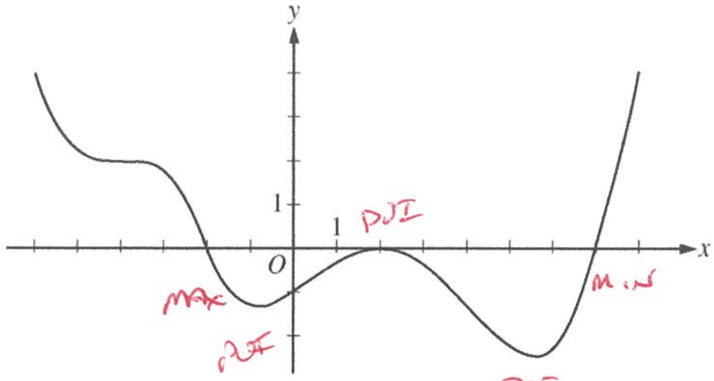
I. $g'(x)$: + 0 − 0 + ; $x$: 0, 3.5

~~II.~~ $g'(x)$: + 0 − dne − 0 + ; $x$: 0, 1, 3.5 $\cdot\, g'(1) = -\pi/2$

~~III.~~ $g''(x)$: + dne − ; $x$: 1 WRONG SIGNS

a) I only b) II only c) I and II only

d) I and III only e) I, II, and III




Graph of $f'$

6. The figure above shows the graph of $f'(x)$, the derivative of function $f$, for $-6 < x < 8$. Of the following, which best describes the graph of $f$ on the same interval?

a) 1 relative minimum, 1 relative maximum, and 3 points of inflection

b) 1 relative minimum, 1 relative maximum, and 4 points of inflection

c) 2 relative minima, 1 relative maximum, and 2 points of inflection

d) 2 relative minima, 1 relative maximum, and 4 points of inflection

e) None of these

7. A particle's acceleration function is $a(t) = \sin 2t$, and its velocity is 0 and position is 1 at $t = 0$. Which of these represents the particle's position function?

a) $x(t) = -\sin 2t + 1$

b) $x(t) = -\sin 2t - t + 1$

c) $x(t) = -\frac{1}{2}\cos 2t + \frac{1}{2}$

d) $x(t) = \frac{1}{2}\cos 2t - \frac{1}{2}$

e) $x(t) = -\frac{1}{4}\sin 2t + \frac{1}{2}t + 1$

$V = \int \sin 2t \, dt = -\frac{1}{2}\cos 2t + C_1$

$X = -\frac{1}{4}\sin 2t + C_1 t + C_2$

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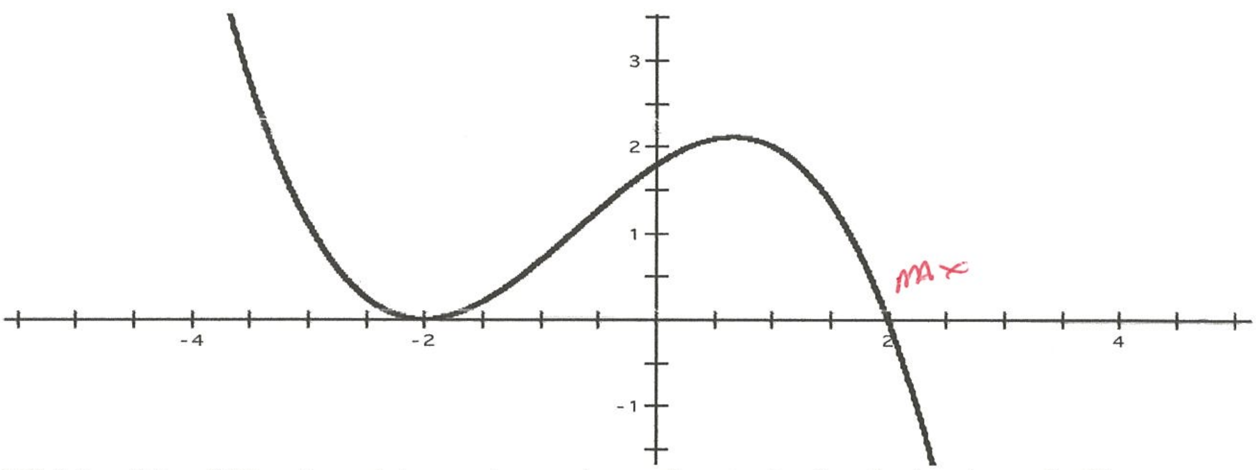
8. The graph of the **second** derivative of $f$ is shown below.




Which of the following statements are **true** about $f'$, the derivative of $f$?

I. The graph of $f'$ has a maximum at $x = 2$.

~~II.~~ The graph of $f'$ is concave down on $x \in (-1, 2)$ $f'$ CON DOWN WHEN $f''$ IS DEC

~~III.~~ The graph of $f'$ is decreasing at $x = -3$. $f'' > 0 \therefore f'$ IS INC

a) I only b) II only c) III only

d) I and III only e) II and III only

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9. What is the minimum value of $f(x) = x^3 - 2x^2$ where $-1 \le x \le 1$?

a) $-3$

b) $-1$

c) $0$

d) $1$

e) No minimum value exists

$f' = 3x^2 - 4x = 0$

$x = 0, 3/4$

$f(3/4) = \frac{27}{64} - \frac{9}{8}$

$= \frac{-45}{64}$

| $x$ | $f(x)$ |
|---|---|
| $-1$ | $-3$ |
| $0$ | $0$ |
| $3/4$ | $-45/64$ |
| $1$ | $-1$ |

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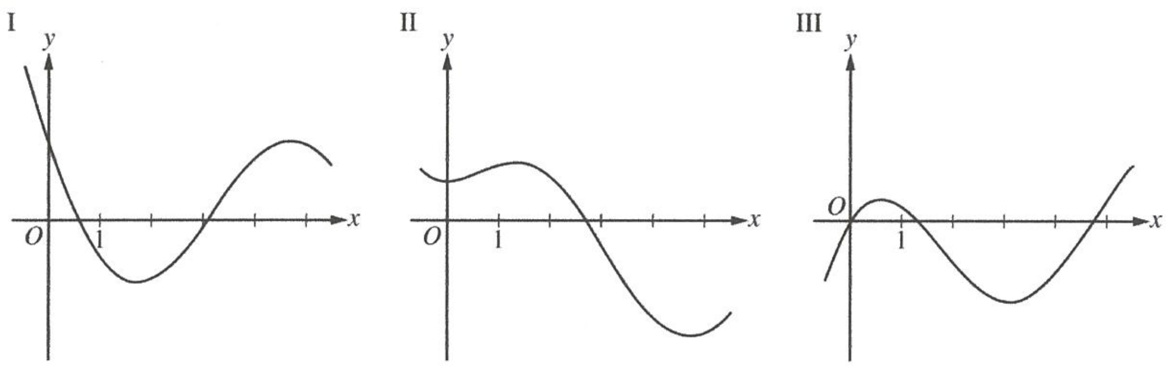


10. Three graphs labeled I, II, and III are shown above. One is the graph of $f(x)$, one is the graph of $f'(x)$, and one is the graph of $f''(x)$. Which of the following correctly identities each of the three graphs?

a) $f(x) = \text{I},\ f'(x) = \text{II},\ f''(x) = \text{III}$
b) $f(x) = \text{II},\ f'(x) = \text{I},\ f''(x) = \text{III}$
c) $f(x) = \text{II},\ f'(x) = \text{III},\ f''(x) = \text{I}$
d) $f(x) = \text{III},\ f'(x) = \text{I},\ f''(x) = \text{II}$
e) $f(x) = \text{III},\ f'(x) = \text{II},\ f''(x) = \text{I}$

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11. Let $H$ represent a circle with diameter $k$. The area of $H$ decreases at a rate of $2\pi$ cm/sec. When the radius is 3cm, what is $\frac{dk}{dt}$ in cm/sec?

a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$ e) 2

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = -2\pi$

$3\frac{dr}{dt} = -1$

$\frac{dr}{dt} = -1/3$

$k = 2r$

$\frac{dk}{dt} = 2\frac{dr}{dt}$

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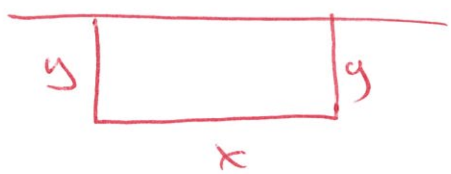
12. A rectangular field is to be enclosed by a fence. An existing fence will form one side of the enclosure. The amount of fence bought for the other three sides is 1200 feet. What is the maximum area of the enclosed field?

a) 160,000 ft$^2$ b) 600 ft$^2$ c) 180,000 ft$^2$

d) 1200 ft$^2$ e) 300 ft$^2$




$A = xy = y(1200-2y)$

$= 1200y - 2y^2$

$x + 2y = 1200$

$A' = 1200 - 4y$

$300 = y$

$A = 300(600) = 18,0000$

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$A = 2(300)(600) = 360000$

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Score__________

Directions: Show all work.

1. Consider the velocity equation $v(t)=\dfrac{5t}{9+t^2}$ on $y(1)=2$.

a) For what values of $t$ is the particle moving down.

$\dfrac{5t}{9+t^2} > 0$

v: $-$ 0 $+$

t: 0

$t \in [0, \infty)$

b) What is the acceleration at $t=3$? Show the derivative work.

$$a = \frac{(9+t^2)(5)-(5t)(2t)}{(9+t^2)^2}$$

$$a(3) = \frac{18(5)-90}{(18)^2} = 0$$

c) Find the particular position equation.

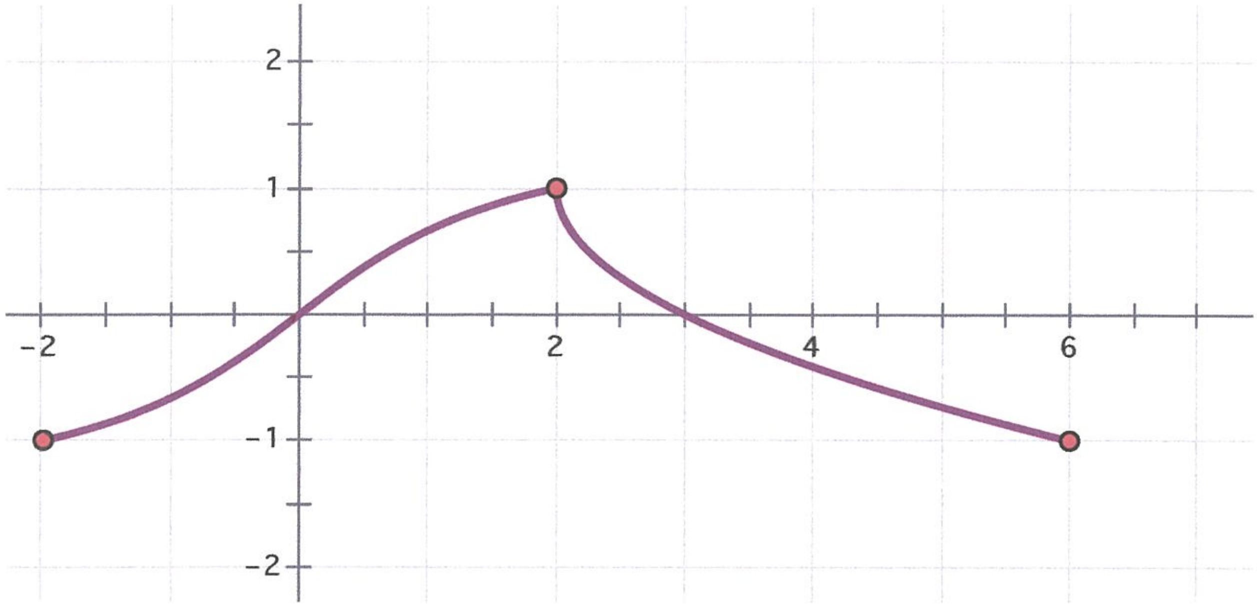
$$y = \int \frac{5t}{9+t^2}\,dt = \frac{5}{2}\int \frac{2t}{9+t^2}\,dt$$

$$= \frac{5}{2}\ln(9+t^2) + c$$

$$2 = \frac{5}{2}\ln 10 + c \rightarrow c = 2 - \frac{5}{2}\ln 10$$

$$y(t) = \frac{5}{2}\ln(9+t^2) + 2 - \frac{5}{2}\ln 10$$

2. Let $g(x)$ be a continuous function on $x \in [-2, 6]$ where the graph of $g'(x)$ is the function shown below.



a) Identify the $x$-value(s) of the relative maximums of $y = g(x)$? Justify your answer.

X=3 BECAUSE $g'$ SWITCHES FROM + TO −

X=−2 BECAUSE −2 IS THE LEFT END POINT AND $g' > 0$ AFTER

b) Identify the $x$-value(s) of the relative minimums of $y = g(x)$? Justify your answer.

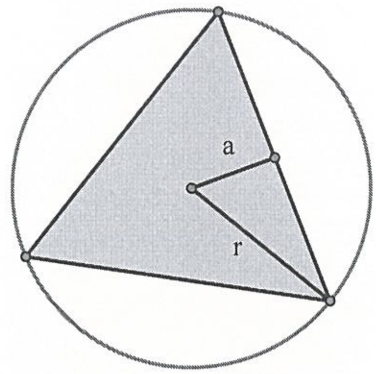
X=0 BECAUSE $g'$ SWITCHES − TO +

X=6 BECAUSE $g'$ IS − BEFORE THE ENDPOINT

c) Where are the points of inflection on $y = g(x)$? Justify your answer.

X=2 BECAUSE $g'$ SWITCHES FROM INCREASING TO DECREASING

3. An equilateral triangle is inscribed in a circle. The circle's circumference is expanding at $6\pi$ in/sec and the triangle maintains the contact of its corners with the circle.




$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 6\pi$

$\frac{dr}{dt} = 3$

Note that the lengths $r$ and $a$ form a 30-60-90 triangle.

a) What is the rate of change of the radius?

$\frac{dr}{dt} = 3$

b) Find the rate of change of the triangle's perimeter when $r = 5$.

$P = 3\sqrt{3}\, r$

$\frac{dP}{dt} = 3\sqrt{3} \frac{dr}{dt} = 9\sqrt{3}$

c) Given that the area of an equilateral triangle is equal to half the apothem $a$ times the perimeter $p$, find out how fast the area inside the circle but outside the triangle is expanding when the area of the circle is $64\pi\ in$. $\rightarrow r = 8$

$A = \frac{1}{2}ap = \frac{1}{2}\left(\frac{r}{2}\right)(3\sqrt{3}\, r) = \frac{3\sqrt{3}}{4} r^2$

TRIANGLE: $\frac{dA_1}{dt} = \frac{3\sqrt{3}}{2} r \frac{dr}{dt}$

CIRCLE: $A = \pi r^2$

$\frac{dA_2}{dt} = 2\pi r \frac{dr}{dt} = 48\pi$

$\left.\frac{dA_1}{dt}\right|_{r=8} = 36\sqrt{3}$

SHADE: $\frac{dA_{SHADE}}{dt} = \frac{dA_2}{dt} - \frac{dA_1}{dt} = \boxed{48\pi - 36\sqrt{3}}$