

AP Calculus BC '19-20

Fall Final Part IIa

Calculator Required

Name:

Solution Key

1. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

a) How many people are in the auditorium when the concert begins?

$$\text{PEOPLE} = \int_0^2 R(t) dt = 980 \text{ PEOPLE}$$

b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

$$R'(t) = 2760t - 2025t^2 = 0$$

$t = 0, \overline{1.363}$

cv	$R(t)$
0	0
1.363	854.527
2	120

c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t . The derivative of w is given by $w'(t) = (2-t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

$$w(2) - w(1) = \int_1^2 w'(t) dt = \int_1^2 (2-t)R(t) dt = 387.5$$

THE TOTAL WAIT TIME FOR THOSE WHO ENTERED THE AUDITORIUM AFTER 1 PM IS 387.5 HRS

d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

$$\frac{1}{950} \int_0^2 (2-t)R(t) dt = .776 \text{ HOURS}$$

ON AVERAGE, A PERSON WAITS .775 HOURS FOR THE SHOW TO START

Distance x	0	60	120	180	240	300	360
Diameter $B(x)$	24	30	28	30	26	24	26

2. A blood vessel is 360 millimeters (mm) long with circular cross-sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at select points along the length of the vessel, where x represents the distance from the end of the vessel and $B(x)$ is a twice-differentiable function that represents the diameter of the blood vessel at that point.

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- a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x=0$ and $x=360$.

$$\frac{1}{360-0} \int_0^{360} B(x) dx =$$

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- b) Approximate the value of your answer in part a) using the data from the table and a Midpoint Riemann sum with three sub-intervals of equal length. Show the computations that lead to your answer.

$$\frac{1}{360} [120 [30 + 30 + 24]] = 14$$

- c) Using correct units, explain the meaning of $\int_{125}^{275} \pi \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

$\frac{B(x)}{2}$ = RADIUS OF THE BLOOD VESSEL AT x AND
 $\pi \left(\frac{B(x)}{2} \right)^2$ IS THE CROSS-SECTIONAL AREA.

$\int_{125}^{275} \pi \left(\frac{B(x)}{2} \right)^2 dx =$ VOLUME IN mm^3 OF THE BLOOD VESSEL
BETWEEN 125 mm AND 275 mm FROM THE END
OF THE VESSEL.

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- d) Explain why there must be at least two values of x , for $0 < x < 360$, such that $B'(x) = 0$.

BECAUSE $B(a) = B(b)$ AT 60 & 180 AND AT 240 & 360,

THERE MUST BE AT LEAST ONE VALUE IN EACH INTERVAL
WHERE $B'(x) = 0$ BY THE MVT.

End of

AP Calculus BC '19-20

Fall Final -- Part IIa

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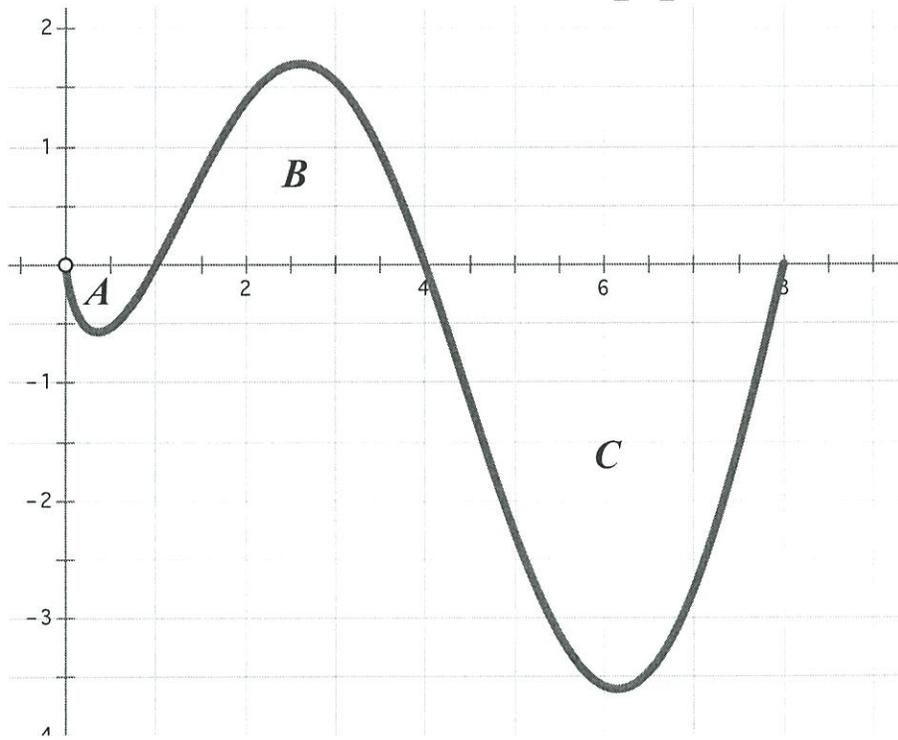
Part IIb

No Calculator Allowed

Name:

3. Below is the graph of $f(x) = (2 \ln x) \sin\left(\frac{\pi}{4}x\right)$, which is defined on

$0 < x \leq 8$. $f(x)$ has horizontal tangent lines at $x = \frac{1}{2}, \frac{5}{2}$, and 6.



Let $A=1$, $B=6$, and $C=14$. A , B , and C represent the areas between $f(x)$ and the x -axis. Let $g(x) = \int_1^x f(t) dt$.

a) Find $g(1)$ and $g'(1)$

$$g(1) = \int_1^1 f(t) dt = 0$$

$$g'(1) = f(1) = 0$$

b) On what intervals, if any, is $g(x)$ increasing and concave down? Explain your reasoning.

g INCREASING means f IS POSITIVE

g CONCAVE DOWN MEANS f IS DECREASING

$$x \in (2.5, 4)$$

c) For $x \in (0, 8]$, find the value of x at which $g(x)$ has an absolute minimum. Justify your answer.

$$g' = f = 0 \text{ AT } x = 0, 1, 4, 8$$

CV	$g(x)$
0	1
1	0
4	6
8	-8

$$\text{ABS MIN} = -8$$

d) Write an expression for $g''(x)$ and find $g''(2)$.

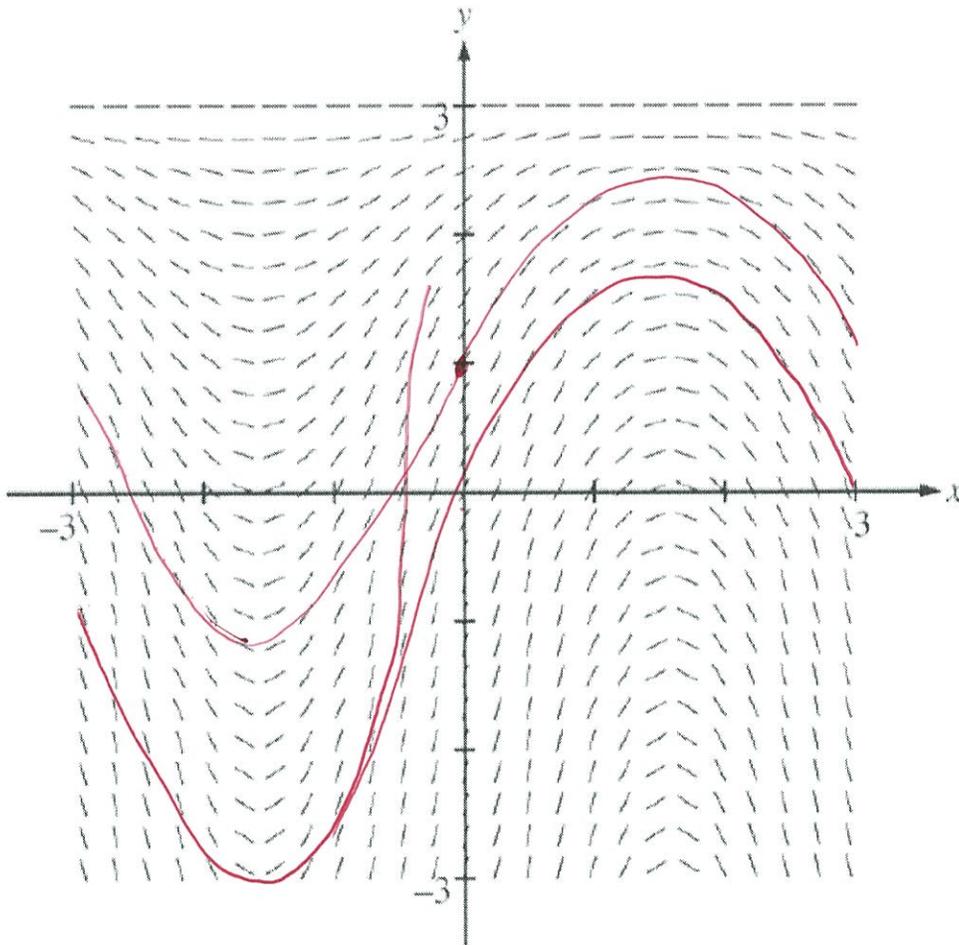
$$g''(x) = f'(x) = \frac{d}{dx} \left[2 \ln x \left| \sin \frac{\pi}{4} x \right. \right] \text{ PRODUCT RULE}$$

$$g''(x) = 2 \ln x \cos\left(\frac{\pi}{4} x\right) \frac{\pi}{4} + \sin\left(\frac{\pi}{4} x\right) \cdot \frac{2}{x}$$

$$g''(2) = 2 \ln 2 \cos\left(\frac{\pi}{4}\right) \left(\frac{\pi}{4}\right) + (1) \cdot 1 = \boxed{1}$$

4. Consider the differential equation $\frac{dy}{dx} = (3-y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function $y = f(x)$ is defined for all real numbers.

a) A portion of the slope field of the differential equation $\frac{dy}{dx} = (3-y)\cos x$ is shown below. Sketch the solution curves through $(-3, -1)$ and $(0, 1)$.



- b) Write an equation for the line tangent to solution curve in part a) at the point $(0,1)$. Use the equation to approximate $f(0.2)$.

$$\left. \frac{dy}{dx} \right|_{0,1} = 2 \cos 0 = 2 \quad y - 1 = 2x$$

$$f(0.2) \approx y = 2(0.2) + 1$$

$$= 1.4$$

- c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$$-\int \frac{1}{3-y} (-dy) = \int \cos x \, dx$$

$$-\ln|3-y| = \sin x + C$$

$$\ln|3-y| = -\sin x + C$$

$$|3-y| = e^{-\sin x + C}$$

$$3-y = Ke^{-\sin x} \quad \rightarrow f(0) = 1 \rightarrow 2 = Ke^0$$

$$2 = K$$

$$3-y = 2e^{-\sin x}$$

$$3 - 2e^{-\sin x} = y$$

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