BC '19-20
Fall Final
Multiple Choice
30 minutes, calculator required

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1. If $f(x) = \ln(\sec x)$, then $f'(x) =$ $\frac{1}{\sec x}$ $\sec x \tan x$

(A) $\tan x$ (B) $\sin x$ (C) $\cos x$ (D) $\sec x$ (E) $\cot x$

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2. $\int (t-4)(t^2-8t)^5\, dt =$ $\frac{1}{2}\int u^5\, du = \frac{1}{12}(t^2-8t)^6 + c$

(A) $\dfrac{(t^2-8t)^6}{6} + C$

(B) $\dfrac{(t^2-8t)^6}{12} + C$

(C) $\dfrac{(t^2-8t)^6}{3} + C$

(D) $\dfrac{(t-4)^6}{6} + C$

(E) $\dfrac{(t-4)^6}{3} + C$

3.

h(t) (°C)	10.5	11.4	12.5	11.3
t (hours)	2	3	5	8

The continuous function h(t) gives the temperature, in °C, of a small town in Finland. Using a trapezoidal sum with subintervals indicated by the table, approximate the average temperature of the town over the interval 2 to 8 hours.

(A) 66.55
(B) 21.85
(C) 11.425
(D) 10.925
(E) 4.50

Trap = $1\left(\frac{11.4+10.5}{2}\right) + 2\left(\frac{11.4+12.5}{2}\right) + \frac{12.5+11.3}{2}(3)$

$10.95 + 23.9 + 35.7$ $3(11.9)$

$\cancel{.9(2.5) + 1.1(4)} +$ $\cancel{\frac{70.55}{6} = 11.758}$

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4. The minimum value of $g(x) = -x^3 + 2x^2$ on $[-1, 3]$ occurs when $x=$

(A) -1 (B) 0 (C) $\frac{4}{3}$ (D) 2 (E) 3

$-3x^2 + 4x = 0$

$x = 0, 4/3$

cv	g(x)
-1	3
0	0
4/3	
3	-9

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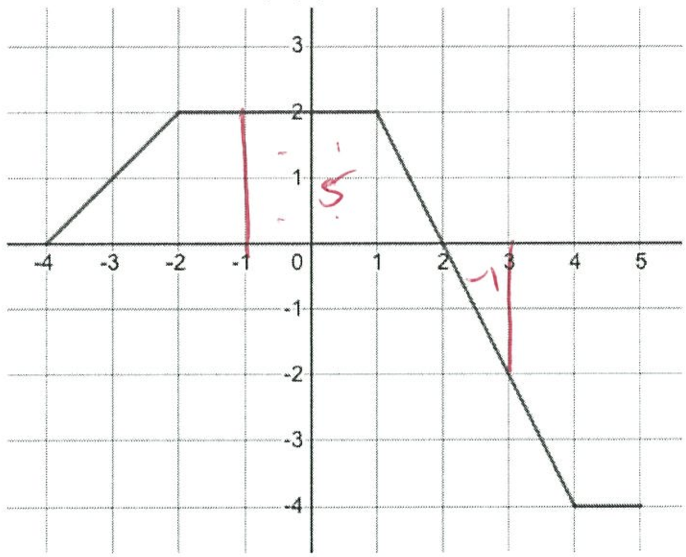
5. The graph below gives the graph of $f'(x)$, the derivative of $f(x)$. If it is known that $f(-1)=3$, what is the value of $f(3)$?



graph of f'(x)

$f(3) = 4$

(A) 3 (B) 4 (C) 6 (D) 7 (E) 9

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6. Using the line tangent to $y=\sqrt[4]{3x}$ at $x=27$, approximate $\sqrt[4]{90}$.

$\frac{dy}{dx} = \frac{1}{4}(3x)^{-3/4}(3) \quad m = \frac{3}{4}\left(\frac{1}{27}\right) = \frac{1}{36}$

$y-3 = \frac{1}{36}(x-27)$

$y = \frac{1}{12}+3$

(A) 3.070

(B) 3.078

(C) 3.080

(D) 3.083

(E) 3.105

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7. Find the total area enclosed by the graphs of $y = -(x-2)^3 + x^2$ and $y = 2x+1$.

$$\int_{.753}^{2.445} (2x+1) - (-(x-2)^3 + x^2)\,dx + \int_{2.445}^{3.802} (-(x-2)^3 + x^2) - (2x+1)\,dx$$

(A) 0.792
(B) 0.987
(C) 1.778
(D) 2.765
(E) 5.306

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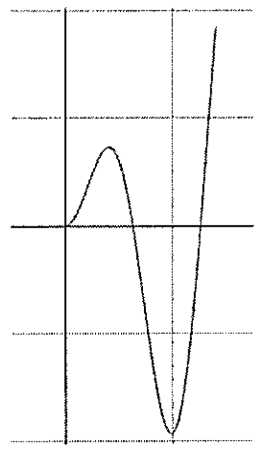
8. The function $k(x)$ is defined below:

$$k(x) = \begin{cases} 12\sin\left(-.4x^4\right), & -2 \le x \le 1 \\ \ln\left(1+e^{2x}\right), & 1 < x \end{cases}$$

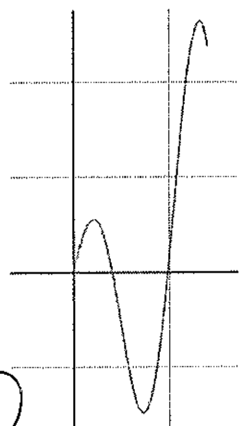
$\int_{-2}^{2} k(x)\,dx =$ $\int_{-2}^{1} 12\sin -.4x^4\,dx + \int_{1}^{2} \ln(1+e^{2x})\,dx$

(A) -8.632
(B) -3.828
(C) -2.210
(D) 0.194
(E) 4.804

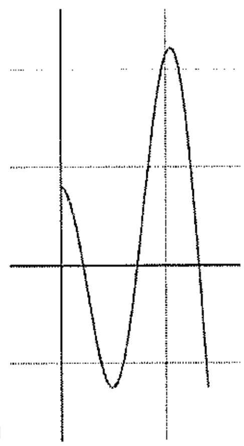
9.



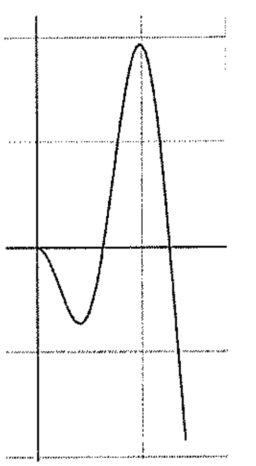
Which of the graphs below represents the derivative of the function graphed above?



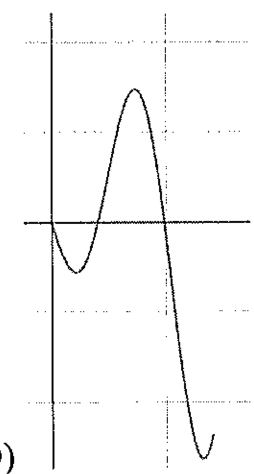
(A)



(B)



(C)



(D)

10. If $h(x)=\int_9^{x^2}\cos^3(\pi t)dt$, then $\lim\limits_{x\to 3}\dfrac{h(x)}{3x-9}=$ L'H $\dfrac{\cos^3 \pi x^2 (2x)}{3} = 2\cos^3 3\pi = -2$

(A) -2
(B) -1
(C) $-\frac{1}{3}$
(D) $-\frac{2}{3}$
(E) DNE

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11. Let **M** represent the region in the first quadrant bounded by $y=2-e^{x^2}$. $\to e^{x^2} = 2-y$
Find the volume of the solid formed by revolving **M** around the **y-axis.**

$x^2 = \ln(2-y)$

$x = \sqrt{\ln(2-y)}$

(A) 0.373

$V=$

(B) 0.386

(C) 1.214

$V = \pi \int_0^1 \left(\sqrt{\ln(2-y)}\right)^2 dy$

(D) 1.712

(E) 1.861

12. $\displaystyle\int_{-\infty}^{-2} \frac{3}{\sqrt[3]{y^5}}\,dy =$ $\quad 3\displaystyle\int_{-\infty}^{-2} y^{-5/3}\,dy = 3\left.\frac{y^{-2/3}}{-2/3}\right]_{-\infty}^{-2}$

$= \frac{9}{2}\, 8^{2/3}$

(A) $\dfrac{-2}{\sqrt[3]{4}}$

(B) $\dfrac{-9}{2\sqrt[3]{4}}$

(C) $\dfrac{3}{\sqrt[3]{4}}$

(D) $\dfrac{3}{\sqrt[5]{4}}$

(E) ∞

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13. Let $y = g(x)$ be a twice-differentiable function and let $y = t(x)$ represent the line tangent to $g(x)$ at $x = 1$. If $t(x) < g(x)$ for all x-values except $x = 1$, which of the following must be true?

TAN LINE BELOW CURVE MEANS CURVE IS CONCAVE UP

(A) $g(1) \geq 0$
(B) $g'(1) \geq 0$
(C) $g'(1) \leq 0$
(D) $g''(1) \geq 0$
(E) $g''(1) \leq 0$

14.

x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	6	2	1
$f'(x)$	9	-4	3	2
$g'(x)$	10	1	3	5

Let $h(x) = g(x) \cdot f(x^3)$. What is the value of $h'(2)$?

(A) -6

(B) 2

(C) 11

(D) 24

(E) 143

$g(2) \cdot f'(8)(3x^2) + f(8) \cdot g'(2)$

$6\,(2)(12) + (-1)(1) =$

$\cancel{288} - 1$

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15. If $j(x)$ is a continuous function where $\int_4^0 j(x)\,dx = 5$, then $\int_4^0 j(4-x)\,dx =$

(A) 5
(B) -5
(C) 11
(D) 21
(E) -4

u

$j(\cancel{4-x})$

$-\int_0^4 j(u)\,du$

16. To which of these functions does the Mean Value Theorem apply on the interval $\left[-1,3\right]$?

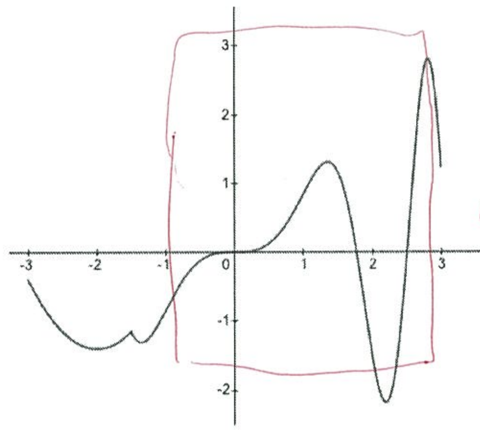
$$f(x)=\begin{cases}4x-1, & -1\le x<2\\ x^2, & 2\le x\le 3\end{cases}$$

NOT CONT

$$g(x)=\begin{cases}3x^4-x, & x\le 1\\ x+1, & x>1\end{cases}$$

$g' = \begin{cases} 12x^3 \\ x \end{cases}$ NOT DIFF

$h(x)$, shown in the graph below



CONT & DIFF

(A) $f(x)$ only (B) $g(x)$ only (C) $h(x)$ only

(D) $g(x)$ and $h(x)$ only (E) None of these