BC Calculus '19-20
Parametric/Polar Test--MC

Name Solution Key

1. Find the equation of the line tangent to the curve determined by

$$\left\{ \begin{array}{c} x=\frac{1}{t} \\ y=\sqrt{t+1} \end{array} \right\} \text{ at } t=3.$$

$x' = \frac{-1}{t^2}$

$y' = \frac{1}{2}(t+1)^{-1/2}$

$x(3) = 1/3$ $x'(3) = -1/9$

$y(3) = 2$ $y'(3) = 1/4$

a) $y-2=-\frac{9}{4}\left(x-\frac{1}{3}\right)$

b) $y-2=-\frac{4}{9}\left(x-\frac{1}{3}\right)$

c) $y-2=\frac{1}{4}\left(x-\frac{1}{3}\right)$

d) $y-\frac{1}{4}=-\frac{4}{9}\left(x+\frac{1}{9}\right)$

e) $y-\frac{1}{4}=-\frac{9}{4}\left(x+\frac{1}{9}\right)$

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2. The total area enclosed by $r=3\cos 3\theta$ is

a) $\frac{7\pi}{4}$ b) $2\pi$ c) $\frac{9\pi}{4}$ d) $\frac{5\pi}{2}$ e) $\frac{11\pi}{4}$

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3. The velocity of a particle's motion is described by $\langle t^2+t-2,\ 2t^2+3t-2\rangle$. At $t=1$, the particle's position is $(3,-5)$. $y(6)=$ $-5+\int_1^6 (2t^2+3t-2)\,dt$

$= -5 +$ MATH 9

a) 79.167

b) 74.167

c) 185.833

d) 180.833

e) 188.833

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4. If $x(t)=5\sin t$ and $y(t)=3\cos t$, then $\dfrac{d^2y}{dx^2}=$

a) $-\dfrac{3}{5}\cot t$ b) $\dfrac{3}{5}\tan t$ c) $-\dfrac{3}{5}\sec^2 t$

d) \$\dfrac{3}{5}\sec^2 t$ e) $-\dfrac{3\sec^3 t}{25}$

$\dfrac{dy}{dx} = \dfrac{-3\sin t}{5\cos t} = \dfrac{-3}{5}\tan t$

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$\dfrac{d^2y}{dt^2} = \dfrac{\frac{d}{dt}\tan t}{5\cos t} = \dfrac{\frac{-3}{5}\sec^2 t}{5\cos t} =$

5. What is the perimeter of one petal of the curve $r = 3\sin 5\theta$?

a) 1.414 b) 6.303 c) 12.606 d) 31.515 e) 63.030

$$\frac{dr}{d\theta} = 3\cos 5\theta\,(5)$$

$$L = \int_0^{\pi/5} \sqrt{(3\sin 5\theta)^2 + (15\cos 5\theta)^2}\, d\theta$$

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6. Give the length of the curve determined by $\left\{ \begin{array}{l} x = 2t - 2\sin t \\ y = 2 - 2\cos t \end{array} \right\}$ on $t \in [0, 2\pi]$

a) $8\sqrt{2}$ b) 16 c) 8

d) $4\sqrt{2}$ e) $16\sqrt{2}$

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7. A particle moves such that its position is described by $\left\{ \begin{array}{c} x = t-1 \\ y = t - e^{-t} - 1 \end{array} \right\}$.

Which of the following statements is true?

I. The particle is moving to the right when t=0

II. The particle is moving up when t=4

~~III.~~ The particle is at rest when t=0

$x' = 1$ ALWAYS RIGHT

$y' = 1 + e^{t}$ ALWAYS UP

a) I only b) II only c) III only

d) I and II only e) II and III only

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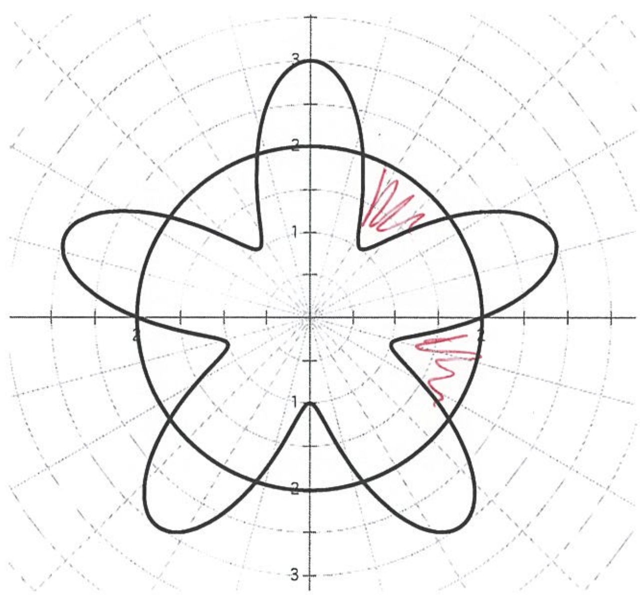

8. The graphs of $r_1 = 2 + \sin(5\theta)$ and $r_2 = 2$ are shown above. What is the total area outside $r_1$ and inside $r_2$?

a) 1.571 b) 1.000 c) 3.142 d) 3.125 e) 4.785

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$2\sin 5\theta = 2$

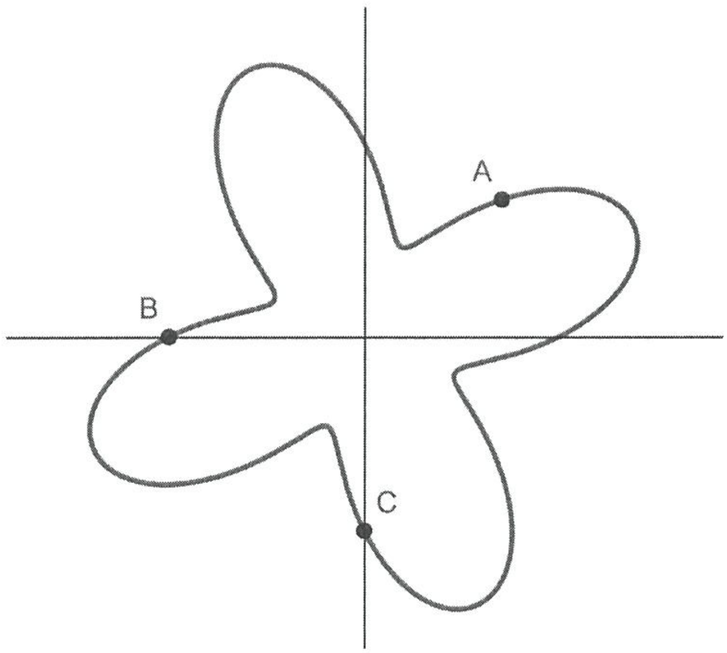
$5\theta = 0 \pm \pi n$

$\theta = 0 \pm \pi/5\, n$

$A = 5 \int_{+\pi/5}^{2\pi/5} \frac{1}{2} (r_2^2 - r_1^2)^2 \, d\theta$

$=$

9. An unknown polar curve $r$ is graphed below. At which of the labeled points would the values of $\frac{dr}{d\theta}$ and $\frac{dy}{dx}$ have opposite signs?




At A $\frac{dr}{d\theta} < 0$ $\frac{dy}{dx} > 0$

At B: $\frac{dr}{d\theta} > 0$ $\frac{dy}{dx} > 0$

At C: $\frac{dr}{d\theta} > 0$ $\frac{dy}{dx} < 0$

a) No points
b) A and B only
c) A and C only
d) B and C only
e) A, B, and C

BC Calculus '19-20 Name Solution Key
Parametric/Polar Test--FR

1. A particle moves in the *xy*-plane so that at time *t*, its position vector is $\langle x(t), y(t)\rangle$. At $t=2$, its position vector is $(1, 5)$. It is also known that $\langle x'(t), y'(t)\rangle = \left\langle \frac{\sqrt{t+2}}{e^t}, \sin^2 t \right\rangle$.

a. Is the particle's horizontal movement to the left or the right at $t=2$? Explain your answer. Find the slope of the particle's path at $t=2$.

2

$x'(2) = \frac{\sqrt{4}}{e^2} > 0 \therefore$ RIGHT

$\frac{dy}{dx} = \frac{\sin^2 2}{2/e^2} = 3.055$

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b. Find the *x*-coordinate of the particle at $t=4$.

2

$x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$

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c. Find the acceleration vector at time $t = 4$.

(2) $\langle -.041, .989 \rangle$

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d. Find the speed at time $t = 4$.

(1) $S = \sqrt{(x'(4))^2 + (y'(4))^2}$

$= \sqrt{.0448^2 + (5.727)^2} = \cancel{.85}\,5.74$

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e. Find the total distance traveled by the particle between $t = 2$ and $t = 4$.

(2) $= \int_2^4 \sqrt{[x'(t)]^2 + [y'(t)]^2} = 0.\cancel{842}\,650$

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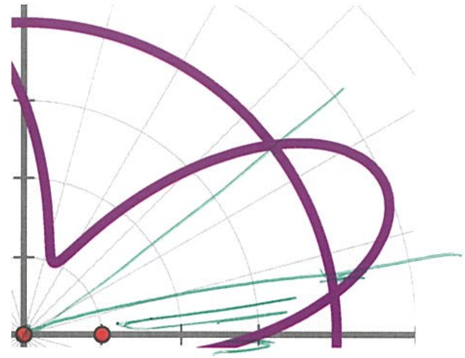
2. The graph below shows the curves $r_1 = 3 + 2\sin 4\theta$ and $r_2 = 4$ in Quadrant I.



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a. Find the points of intersection of the two curves. Show the algebra.

$3 + 2\sin 4\theta = 4$

$\sin 4\theta = 1/2$

(2)

$4\theta = \begin{cases} \pi/6 \pm 2\pi n \\ 5\pi/6 \pm 2\pi n \end{cases}$

$\theta = \frac{\pi}{24}, \frac{5\pi}{24}$

$\theta = \begin{cases} \pi/24 \pm \frac{\pi}{2} n \\ 5\pi/24 \pm \frac{\pi}{2} n \end{cases}$

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b. Set up, but do not evaluate, an expression involving one or more integrals that would find the area inside **both** $r_1 = 3 + 2\sin 4\theta$ and $r_2 = 4$ in Quadrant ~~2.~~ I

$\cancel{A = 2\int_{\pi/24}^{5\pi/24} \frac{1}{2}(3 + 2\sin 4\theta)^2 d\theta - \frac{1}{2}\int_{\pi/24}^{5\pi/24} 4^2 d\theta}$

(2)

$A = \int_0^{\pi/24} \frac{1}{2}(3 + 2\sin 4\theta)^2 d\theta + \frac{1}{2}\int_{\pi/24}^{5\pi/24} 16\, d\theta + \frac{1}{2}\int_{5\pi/24}^{\pi/2} (3 + 2\sin 4\theta)^2 d\theta$

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c. Find the value of $\theta$ that corresponds to the point on $r_1 = 3 + 2\sin 4\theta$ that is closest to the pole. Justify your answer.

$$\frac{dr}{d\theta} = 8\cos 4\theta = 0$$

$$\cos 4\theta = 0$$

$$4\theta = \pm\frac{\pi}{2} \pm 2\pi n$$

3

$$\theta = \pm\frac{\pi}{8} \pm \frac{\pi}{2}n$$

$$= \frac{\pi}{8},\ \cancel{\frac{5\pi}{8}}\ \frac{3\pi}{8}$$

| $\theta$ | $r$ |
|---|---|
| 0 | 3 |
| $\pi/8$ | 5 |
| $3\pi/8$ | 1 |
| $\pi/2$ | 3 |

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d. For the curve $r_1$, write an expression for $\frac{dy}{dx}$ in terms of $\theta$.

2

$$\frac{dy}{dx} = \frac{\frac{d}{d\theta} r\sin\theta}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + \sin\theta\frac{dr}{d\theta}}{-r\sin\theta + \cos\theta\frac{dr}{d\theta}}$$

$$= \frac{(3+2\sin 4\theta)\cos\theta + \sin\theta\,(8\cos 4\theta)}{-(3+2\sin 4\theta)\sin\theta + \cos\theta\,(8\cos 4\theta)}$$

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**Q2Q COMICS #532: MEASURE TWICE**

February 12, 2020