AP Calculus BC '19-20
Volume Test
Calculator Allowed

Name: Solution Key

Score: ______________

1. What is the area enclosed by $y = \ln(2x+1)$ and $y = 2\sin x$?

a) 0.334

b) 0.661

c) 3.526

d) 0.825

e) 2.983

$$\int_0^{2.154} \left[2\sin x - \ln(2x+1)\right] dx$$

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2. A region is bounded by $y = \frac{2}{\sqrt{x}}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. The area of this region:

a) is independent of m.
b) increases as m increases.
c) decreases as m increases.
d) increases until $m = \frac{1}{2}$, then decreases.
e) is none of the above

$$A = \int_m^{2m} \frac{2}{\sqrt{x}}\, dx$$

$$= 4\sqrt{x}\Big]_m^{2m}$$

$$= 4\sqrt{2m} - 4\sqrt{m}$$

$$= 4(\sqrt{2}-1)\sqrt{m}$$

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3. Let R be the region in the first quadrant bounded by $x = \sin^{-1} y$, the x-axis, and $x = \frac{\pi}{2}$. Which of the following integrals gives the volume of the solid generated when R is rotated about the line $x = \frac{\pi}{2}$?

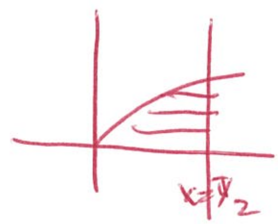
a) $\pi \int_0^{\pi/2} y^2 \, dy$

b) $\pi \int_0^1 \left(\frac{\pi}{2} - \sin^{-1} y\right)^2 dy$

c) $\pi \int_0^{\pi/2} \left(\sin^{-1} y\right)^2 dy$

d) $\pi \int_0^1 (\sin x)^2 \, dx$

e) $\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - \sin x\right)^2 dx$



DISK

$x = \sin^{-1} y$

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4. Which of the following integrals gives the length of the graph $y = \sin 2x$ from $x=a$ to $x=b$?

$\frac{dy}{dx} = \cos 2x \, (2)$

a) $\int_a^b \sqrt{1 + 4\sin^2 x \cos x} \, dx$

b) $\int_a^b \sqrt{1 + \sin^2 2x} \, dx$

c) $\int_a^b \sqrt{1 + 4\cos^2 2x} \, dx$

d) $\int_a^b \sqrt{1 + \cos^2 2x} \, dx$

e) $\int_a^b \sqrt{1 + 4\sin^2 x \cos^2 x} \, dx$

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)\,dx$?

a) The total change in temperature during the first t hours.
b) The total change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24^{th} day.

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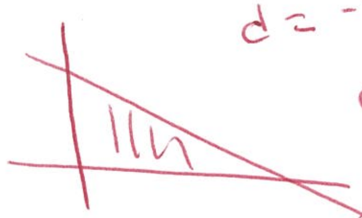
6. Let $\boldsymbol{R}$ represent the region in the first quadrant bounded by $y = -3x + 6$. Which expression gives the volume of the solid with base $\boldsymbol{R}$ whose cross-section perpendicular to the x-axis are semi-circles?

a) $\pi \int_0^2 (-3x+6)^2\, dx$

$d = -3x + 6$

$r = \frac{-3}{2}x + 3$



b) $\pi \int_0^6 \left(\frac{6-y}{3}\right)^2 dy$

c) $\frac{\pi}{4} \int_0^2 (-3x+6)^2\, dx$

d) $\frac{\pi}{8} \int_0^2 (-3x+6)^2\, dx$

e) $\frac{\pi}{2} \int_0^6 \left(\frac{6-y}{6}\right)^2 dy$

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7. What is the volume of the solid formed when the region bounded by $y = e^x - 1$, $y = 2$, and $x = 0$ is rotated around the y-axis?

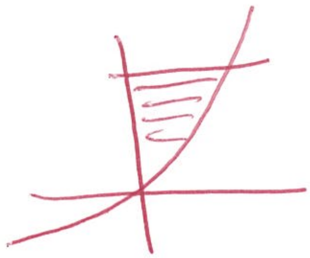
a) 1.296

b) 3.233

c) 5.930

d) 10.354

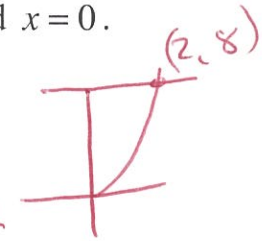
e) 25.199



$x = \ln(y+1)$

$V = \pi \int_0^2 \ln^2(y+1)\, dy =$

8. Let S be the region bounded by $y = x^3$, $y = 8$, and $x = 0$.



a) Find the area of S. Show the setup.

$$A = \int_0^2 (8 - x^3)\,dx = \left[8x - \frac{x^4}{4}\right]_0^2$$

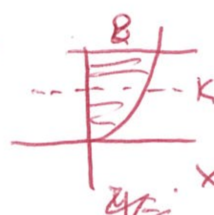
$$= \cancel{37.699} \qquad = 12$$

b) Find the volume of the solid generated by revolving S around the x-axis. Show the anti-differentiation steps.

$$V = \pi \int_0^2 8^2 - (x^3)^2\,dx = \pi\left[64x - \frac{1}{7}x^7\right]_0^2$$

$$\approx 344.678$$

c) There exists a number k, $0 < k < 8$, such that when the part of S below the line $y = k$ is revolved around the y-axis, the resulting solid has the same volume as the solid formed by when the part of S above the line $y = k$ is revolved around the y-axis. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

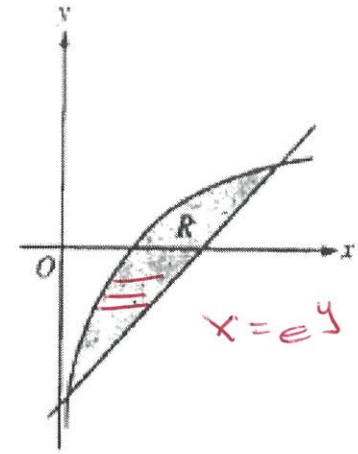


$\cancel{V = \pi\int_{-2}^{2} (\ldots)}$

$x = y^{1/3}$

$$V = \pi\int_0^k y^{2/3}\,dy = \pi\int_k^8 y^{2/3}\,dy$$

9. Let $\boldsymbol{R}$ be the region bounded by $y = \ln x$ and $y = x - 2$.



$x = e^y$ $\quad$ $x = y + 2$

a) Find the volume of the solid whose base is $\boldsymbol{R}$ and whose cross-sections perpendicular to the y-axis are isosceles right triangles with the one leg is in the base.

$A = \frac{1}{2}s^2$ WHERE $s = (y+2) - e^y$

3

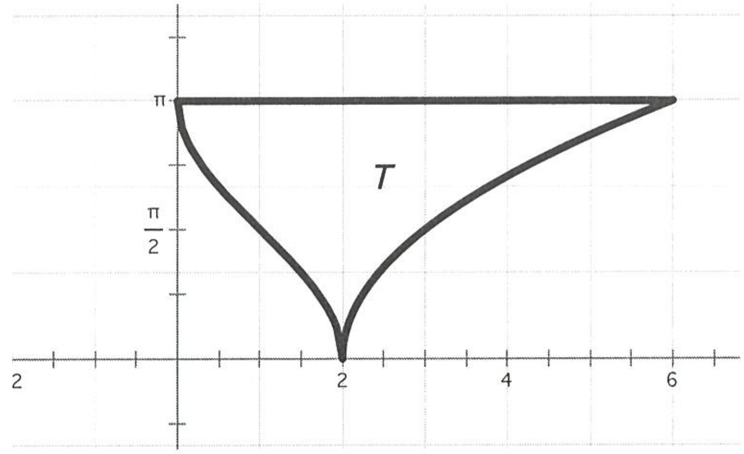
$$V = \int_{-1.841}^{1.146} \frac{1}{2}\left[y+2-e^y\right]^2 dy = .773$$

4

b) Find the volume of the solid formed by revolving $\boldsymbol{R}$ around the line $y = 3$.

$$V = \pi \int_{.158}^{3.146} (3-(x-2))^2 - (3 - \ln x)^2 \, dx$$

$= 39.280$



10. Let ***T*** be the region above bounded by

$$y = \frac{\pi}{2} - \sin^{-1}(x-1), \quad y = \frac{\pi}{2}\sqrt{x-2}, \text{ and } y = \pi$$

$x = 1 + \sin\left(\frac{\pi}{2} - y\right)$ $\qquad x = \frac{4}{\pi^2}y^2 + 2$

4 a) Find the perimeter of ***T***. Show the set-up.

$$P = 6 + \int_0^2 \sqrt{1 + \left(\frac{1}{2x - x^2}\right)}\, dx$$

$$+ \int_2^6 \sqrt{1 + \frac{\pi^2}{16(x-2)}}\, dx$$

$= 6 + 3.820 + 5.324$

$= 15.144$

$$\frac{dy_1}{dx} = -\frac{1}{\sqrt{1-(x-1)^2}}$$

$$= \frac{-1}{\sqrt{2x - x^2}}$$

$$\frac{dy_2}{dx} = \frac{\pi}{4}(x-2)^{-1/2}$$

$L_1 =$

$L_2 =$

b) Find the volume of the solid formed by revolving ***T*** around the y-axis.

4 $$V = \pi \int_0^{\pi} \left(\frac{4}{\pi^2}y^2 + 2\right)^2 - \left(1 + \sin\left(\frac{\pi}{2} - y\right)\right)^2 dy$$

$= 108.894$