AP Calculus BC '25-26

Dr Quattrin

Fall Final Part I

Calculator Allowed
45 minutes

Name:

Solutial Key

1. Which of the following statements must be false?

a) $\frac{d}{dx}\left(4e^{\sec 3x}\right) = 4e^{\sec 3x}\sec 3x\tan 3x$ F

b) $\frac{d}{dx}\left(\sin^{-1}2x^3\right) = \frac{6x^2}{\sqrt{1-4x^6}}$

c) $\frac{d}{dx}\left(\ln\left(2x^2+4x-5\right)\right) = \frac{4x+4}{2x^2+4x-5}$

d) $\frac{d}{dx}\left(\sqrt{\tan\left(1-x^2\right)}\right) = \frac{-x\sec^2\left(1-x^2\right)}{\sqrt{\tan\left(1-x^2\right)}}$

---

2. If $h(t) = t^2 \sin 3t$, then $h'\left(\frac{\pi}{2}\right)$

a) $-\pi$ b) $-\frac{3\pi^2}{4}$ c) 0 d) $\frac{3\pi^2}{4}$ e) π

$h' = t^2(\cos 3t)(3) + 2t\sin(3t)$

$= \left(\frac{\pi}{2}\right)^2 \cos\frac{3\pi}{2}(3) + \pi\sin\frac{3\pi}{2}$

$= -\pi$

$\int_{-2}^{1} + \int_{1}^{5} = \int_{-2}^{5} \rightarrow \int_{-2}^{1} - 3 = -2$ $\quad \int_{-2}^{1} = 1$

$\int_{-2}^{5} f(x)dx = -2$	$\int_{1}^{5} f(x)dx = -3$
$\int_{-2}^{1} g(x)dx = -4$	$\int_{5}^{1} g(x)dx = 9$

3. Based on the information above, $\int_{1}^{-2}[g(x)+f(x)]dx =$

a) -9 b) -1 c) ~~0~~ d) 3 e) 9

$$= \int_{1}^{-2} g(x)\,dx + \int_{1}^{-2} f(x)\,dx$$

$$= 4 + (-1) = 3$$

---

4. A particle travels along a straight line with a velocity of $v(t) = 3e^{-t}\sin(2t)$ meters per second. What is the average distance traveled, in meters per second per second, of the particle during the time interval $0 \le t \le 2$ seconds?

$$\frac{1}{2-0}\int_{0}^{2} |v(t)|\,dt$$

(a) 1.531

(b) 1.368

(c) 0.684

(d) 0.766

---

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	3	4	-1
4	2	-1	8	7
16	4	2	2	4

5. Selected values of f, g, and their derivatives are indicated in the table above. Let $B(x) = f(\sqrt{x}) \cdot g(x)$. What is the value of $B'(4)$?

a) 8 b) 10 c) ~~36~~ 3 d) 44

$f'(2) \cdot g'(4) + g(4) f'(2) \frac{1}{2\sqrt{4}}$

$= 4(7) + 8(3)\left(\frac{1}{4}\right) = 27 + 6 = 33$

---

6. If $f(x) = x \ln^2 x$, then $f''(2) =$

a) $2\ln 2 + \ln^2 2$ b) $1 + \ln 2$

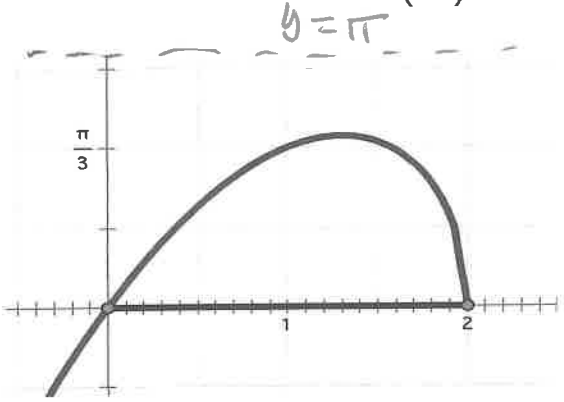
c) $1 + 2\ln 2$ d) $4\ln 2 + \ln^2 2$

$f'(x) = \frac{2x \ln x}{x} + \ln^2 x = 2\ln x + \ln^2 x$

$f''(x) = \frac{2}{x} + 2\frac{\ln x}{x}$

---

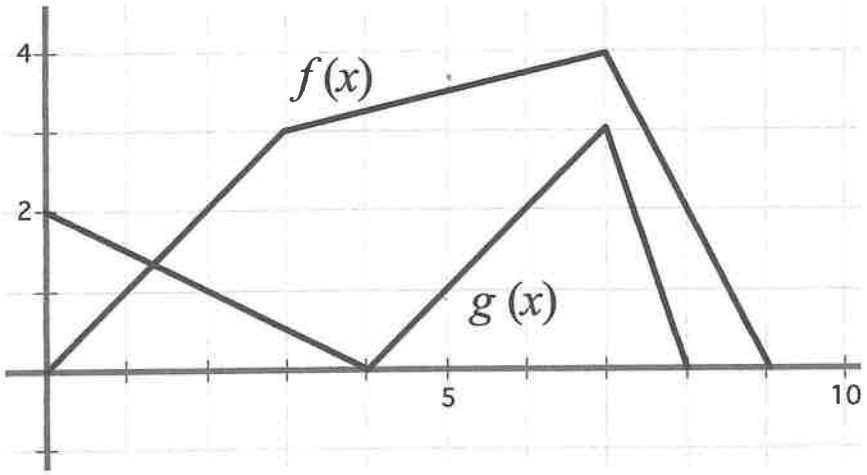
7. Let Q be the region bounded by $y = x\cos^{-1}\left(\frac{x}{2}\right)$, and $x = 0$.



Which of the following will determine the volume of the solid formed by region Q being revolved about the line $y = \pi$?

(a) 4.514 (b) 18.302 (c) 26.492 (d) ~~35.520~~ 48.883

$$V = \pi \int_0^2 \pi^2 - \left(\pi - x\cos^{-1}\frac{x}{2}\right)^2 dx$$



8. Let $f(x)$ and $g(x)$ be linear differentiable functions defines by the graphs above. If $h(x) = \dfrac{g(x)}{f(x)}$, what is the value of $h'(5)$?

(a) $-\dfrac{13}{49}$ (b) $-\dfrac{13}{4}$ (c) $\dfrac{13}{4}$ (d) $\dfrac{13}{49}$

$$h'(5) = \frac{f(5)\,g'(5) - g(5)\,f'(5)}{[g(5)]^2}$$

$$= \frac{(3.5)(1) - 1\left(\frac{1}{4}\right)}{(3.5)^2} = \frac{\frac{7}{2} - \frac{1}{4}}{} = \frac{\frac{13}{4}}{\frac{49}{4}}$$

9. $\int \left(\dfrac{52w^{7/2} - 66w^{5/2} + 22w^{3/2}}{\sqrt{w}} \right) dw =$ $\int (52w^3 - 66w^2 + 22w)\, dw$

~~a)~~ $\left(\dfrac{2}{9}w^{9/2} - \dfrac{132}{7}w^{7/2} + \dfrac{44}{5}w^{5/2} \right)\left(2w^{1/2}\right) + c$

~~b)~~ $52w^3 - 66w^2 + 22w + c$

c) $13w^4 - 33w^3 + 11w^2 + c$

d) $13w^4 - 22w^3 + 11w^2 + c$

---

10. If $y = x + \sin(xy)$, then $\dfrac{dy}{dx} =$ $1 + \cos(xy)\left[x\dfrac{dy}{dx} + y\right]$

(a) $1 + \cos(xy)$ (b) $1 + y\cos(xy)$ (c) $\dfrac{1}{1 - \cos(xy)}$

(d) $\dfrac{1}{1 - x\cos(xy)}$ (e) $\dfrac{1 + y\cos(xy)}{1 - x\cos(xy)}$

$\dfrac{dy}{dx} = 1 + x\cos(xy)\dfrac{dy}{dx} + y\cos xy$

$(1 - x\cos xy)\dfrac{dy}{dx} = 1 + y\cos(xy)$

---

11. One innovative approach to global warming is to capture carbon dioxide that is created as a byproduct of producing concrete and storing the CO_2 in the sandstone in depleted natural gas fields. At a particular site on a particular day, the CO_2 is injected into the sandstone at a rate of $I(t) = 121\sin\left[\frac{\pi}{65}t^2\right]$. The CO_2 stabilizes the ground and forces remaining natural gas upward where it can be extracted at a rate of $E(t) = 50 - 50\cos\left[\frac{\pi}{8}t\right]$. $I(t)$ and $E(t)$ are measured in metric tons per hour and t is measured in hours where $0 \le t \le 8$. Which of the following best describes the equation $\frac{1}{8}\int_0^8 [I(x) - E(x)]\,dx$? AVERAGE VALUE

(a) The total change of gas stores in the sandstone during these eight days.

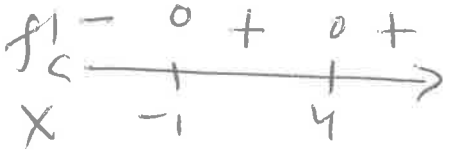
(b) The total amount of gas stored in the sandstone at the end of Day Eight.

(c) The average rate of the stored gas during these eight days.

(d) The average amount of gas stored per day during these eight days.

---

12. Suppose $f'(x) = \frac{(x+1)^3(x-4)^2}{(x^4+1)}$. Which of the following statements must be true?



(a) The slope of the line tangent to $y = f(x)$ at $x = 1$ is 36. F

(b) $f(x)$ is increasing on $x \in (1, 4)$ F

(c) $f(x)$ has a minimum at $x = -1$ F

(d) Two of these are true.

(e) All of these are true.

---

13. Let $y = f(x)$ be a twice-differentiable function such that $y(0) = 1$ and $\frac{dy}{dx} = \frac{4x}{2-y}$. The particular solution is

(a) $f(x) = 2 + \sqrt{1-4x^2}$

(b) $f(x) = 2 - \sqrt{1-4x^2}$

(c) $f(x) = 2 + \sqrt{4x^2-1}$

(d) $f(x) = 2 - \sqrt{4x^2-1}$

$(2-y)\,dy = 4x\,dx$

$-\frac{(2-y)^2}{2} = 2x + C \qquad (2-y)^2 = C - 4x$

---

14. If h is the function defined by $h(x) = e^{4x} - 9x + 2$, what is the equation of the tangent line to the function when $h'(x) = -5$?

a) $y = -5x + 2$

b) $y = -5(x-2)$

c) $y = -5x$

d) $y = -5x + 3$

e) $y = -5(x-3)$

~~$y = -2x + 5$~~

$h' = e^{4x}(4) - 9 = -5$

$e^{4x} = 1$

$4x = \ln 1 = 0$

$x = 0 \rightarrow y = 3$

---

15. This problem involves finding the absolute maximum and absolute minimum of the function $f(x)=x^3-3x+4$ restricted to the closed interval $x\in[0,2]$. Which of the following statements is correct?

(a) $f(x)$ has both an absolute maximum and absolute minimum at the end points.

(b) $f(x)$ has both an absolute maximum and absolute minimum at interior points.

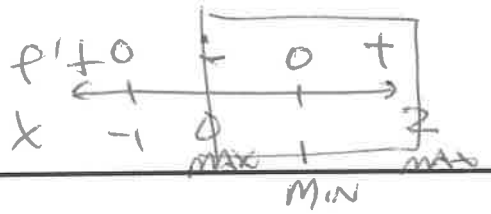
(c) $f(x)$ has both an absolute maximum at an end point and an absolute minimum at an interior point.

(d) $f(x)$ has both an absolute maximum at an interior point and an absolute minimum at an end point.

(e) None of the above

$f'(x)=3x^2-3=0$

$x=\pm 1$



Time t in seconds	0	1.1	2	3.74	4.82	6.18	7.51	9.19	11.95
$v(t)$ in yards/sec	0	4.38	6.24	9.75	11.28	10.52	8.35	7.02	4.62

16. The table above lists approximate velocities achieved by running back Raheem Mostert as he set a record as the ballcarrier to hit the highest velocity in a game vs the New York Jets in 2020. Using a trapezoidal sum, approximately how many yards had Mostert gained at ~~$t=2$~~ From $t=0$ to $t=2$

a) 7.19 b) 8.76 c) 10.43 d) 12.48

$$\int_0^2 v(t)\,dt \approx 1.1(2.19)+.9(5.31)$$

17. A cup of coffee is made with boiling water at a temperature of 100 C°, in a room at temperature 20 C°. After two minutes, it has cooled to 80 C°. According to Newton's Law of Cooling, the temperature of the coffee follows the differential equation

$$\frac{dy}{dt} = -0.144(y-20),$$

where y is the temperature of the coffee at time t minutes. Which of the following statements would be true?

a) $y(t) = -0.072(y-2)^2 + c$

b) $\ln(y-20) = -0.144t + c$

c) $\ln(y-20) = 0.144t + c$

d) $\frac{1}{2}(y-20)^2 = -0.144t + c$

$\frac{1}{y-20}\,dy = -0.144$

$\ln|y-20| = -.144t + C$

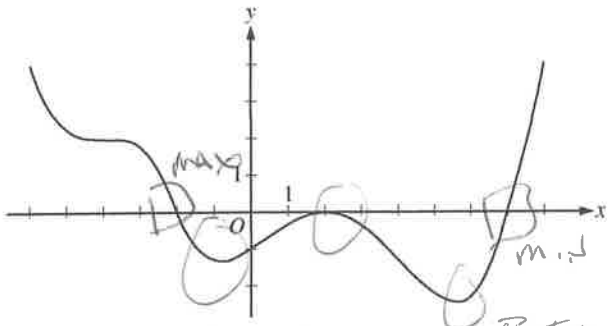
---

18. $\int_0^{\frac{\pi}{3}} \sec x \tan x(1+\sec x)\,dx$

$u = \sec x + 1$

a) 4 b) $\frac{5}{2}$ c) 3 d) $\frac{9}{2}$ e) 5

$\int_2^3 u\,du = \frac{u^2}{2}\Big]_2^3 = \frac{9}{2} - \frac{4}{2}$



Graph of f'

19. The figure above shows the graph of $f'(x)$, the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

a) 1 relative minimum, 1 relative maximum, and 3 points of inflection

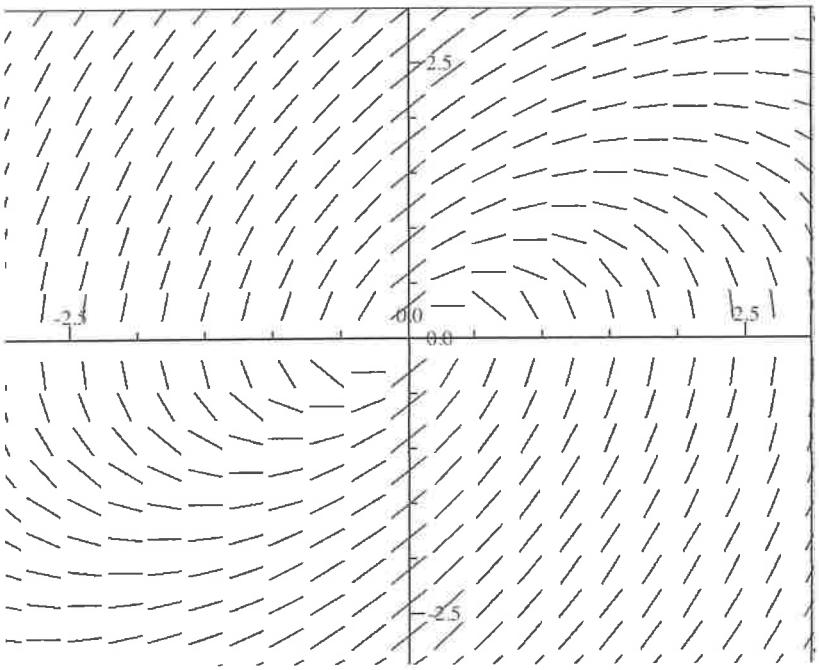
b) 1 relative minimum, 1 relative maximum, and 4 points of inflection

c) 2 relative minima, 1 relative maximum, and 2 points of inflection

d) 2 relative minima, 1 relative maximum, and 4 points of inflection

e) 2 relative minima, 2 relative maxima, and 3 points of inflection

20. Which of the following equations corresponds to the solution to the slope field shown in the figure below?

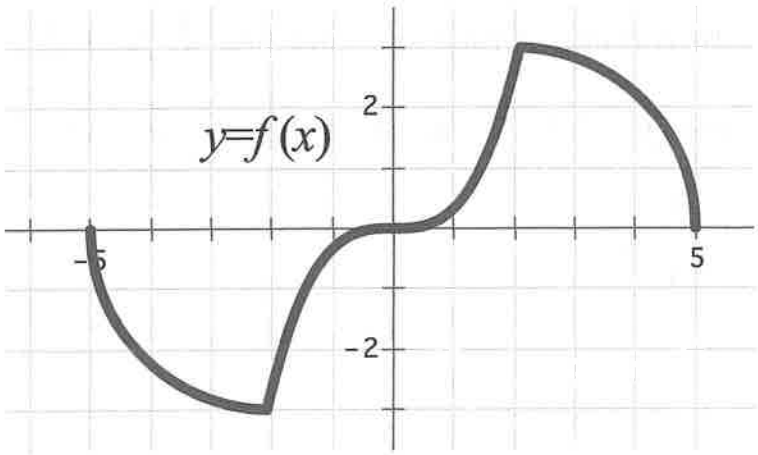


a) $\frac{dy}{dx} = 1 - \frac{x}{y}$ $\quad x=0 \rightarrow \frac{dy}{dx} = 1$

b) $\frac{dy}{dx} = y - x$

c) $\frac{dy}{dx} = -\frac{x^2}{y^2}$

d) $\frac{dy}{dx} = -\frac{y}{x}$



21. The graph of the function $f(x)$ is shown above. If $h(x) = \int_x^5 f(t)\,dt$, on what interval is $h(x)$ increasing and concave up?

(a) (-5, -2) (b) (-2, 0) (c) (0, 2) (d) (2, 5)

$h' = -f$ ∴ h IS INC & CON UP IF

f IS NEGATIVE & DECREASING
~~POSITIVE & INCREASING~~

End of

AP Calculus BC '25-26

Fall Final Part I