A. P. CALCULUS BC FORMULA BOOKLET

GRAPHING CALCULATORS

Each student will be expected to bring to the examination a graphing calculator on which the student can:

1. produce the graph of a function within an arbitrary viewing window;
2. find the zeros of a function;
3. compute the derivative of a function numerically, and
4. compute definite integrals numerically.

Pay special attention to calculator syntax; i.e., placement of parentheses, commas, variables, and order of operations. Important functions include ***graph, root, solve, nDeriv,*** and ***fnInt.***

CALCULATORS should be in **RADIAN MODE.**

CONTINUITY: The function $f(x)$ is said to be continuous at $x = c$ if

1) $f(c)$ is a finite number;
2) $\lim_{x \to c} f(x)$ exists;
3) $\lim_{x \to c} f(x) = f(c)$.

DIFFERENTIABILITY: The function $f'(x)$ is continuous at $x = c$.

DIFFERENTIABILITY IMPLIES CONTINUITY,
BUT CONTINUITY DOES NOT IMPLY DIFFERENTIABILITY.

LIMITS: ZEROS IN NUMERATOR/DENOMINATOR OF A FRACTION
("c" is a constant.)

Zero (Root)	$\frac{0}{c} = 0$
Vertical Asymptote	$\frac{c}{0} = \pm\infty$ (= D.N.E.)
Point of Exclusion (Removable Discontiuity)	$\frac{0}{0}$ (= undefined)

DERIVATIVE OF A FUNCTION: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

DIFFERENTIATION RULES:
(Where "**u**" and "**v**" are differentiable functions of x, and "**a**" is a constant.)

$\frac{d}{dx}au=a\bullet\frac{du}{dx}$

$\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$

$\frac{d}{dx}u^n=nu^{n-1}\bullet\frac{du}{dx}$

$\frac{d}{dx}a=0$

$\frac{d}{dx}(uv)=u\bullet\frac{dv}{dx}+v\bullet\frac{du}{dx}$

$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\bullet\frac{du}{dx}-u\bullet\frac{dv}{dx}}{v^2}$

CHAIN RULE: $\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx}$

$\frac{d}{dx}\sin u=\cos u\ \frac{du}{dx}$

$\frac{d}{dx}\cos u=-\sin u\ \frac{du}{dx}$

$\frac{d}{dx}\tan u=\sec^2 u\ \frac{du}{dx}$

$\frac{d}{dx}\cot u=-\csc^2 u\ \frac{du}{dx}$

$\frac{d}{dx}\sec u=\sec u\tan u\ \frac{du}{dx}$

$\frac{d}{dx}\csc u=-\csc u\cot u\ \frac{du}{dx}$

$\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}e^u=e^u\ \frac{du}{dx}$

$\frac{d}{dx}a^u=a^u\ln a\ \frac{du}{dx}$

$\frac{d}{dx}\sin^{-1}u=\frac{\frac{du}{dx}}{\sqrt{1-u^2}}$

$\frac{d}{dx}\cos^{-1}u=\frac{-\frac{du}{dx}}{\sqrt{1-u^2}}$

$\frac{d}{dx}\tan^{-1}u=\frac{\frac{du}{dx}}{1+u^2}$

$\frac{d}{dx}\cot^{-1}u=\frac{-\frac{du}{dx}}{1+u^2}$

$\frac{d}{dx}\sec^{-1}u=\frac{\frac{du}{dx}}{|u|\sqrt{u^2-1}}$

$\frac{d}{dx}\csc^{-1}u=\frac{-\frac{du}{dx}}{|u|\sqrt{u^2-1}}$

RELATIVE EXTREMA:

Critical Value (x-coordinate of an Extreme)

a is a critical value of f(x) iff f(a) is in the doamin and

i) $f'(a)=0$

ii) $f'(a)$=does not exist

or iii) a is an endpoint of the domain of f(x)

1st Derivative Test

$f'(a) = 0$ and sign pattern switches - to + ⇒⇒⇒ f(a) is a relative minimum .

$f'(a) = 0$ and sign pattern switches + to - ⇒⇒⇒ f(a) is a relative maximum.

$f'(a) = 0$ and sign pattern does not switch ⇒⇒⇒ f(a) is not an extreme.

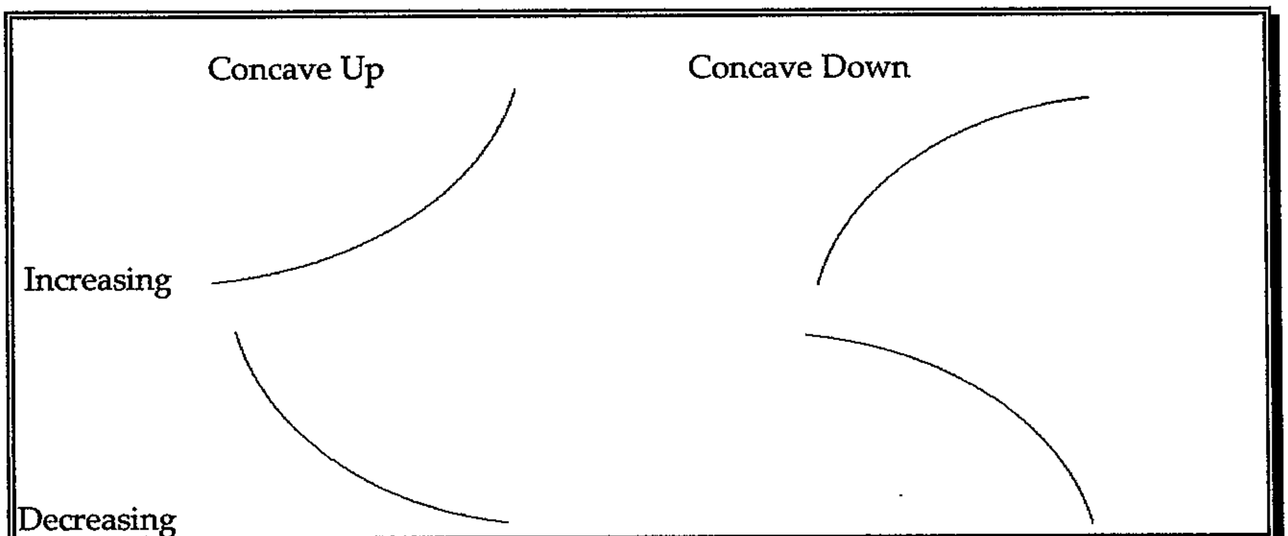
2nd Derivative Test

$f'(a) = 0$ and $f''(a) > 0$ ⇒⇒⇒ f has a relative minimum at $x = a$.

$f'(a) = 0$ and $f''(a) < 0$ ⇒⇒⇒ f has a relative maximum at $x = a$.

Conclusion Chart

Signf(x)		f'(x)	f''(x)
+	Curve above x-axis	Increasing	Concave Up
0	x-intercept (zero)	Critical value	POI
-	Curve below x-axis	Decreasing	Concave Down



VELOCITY:

$$V = \frac{ds}{dt}$$

i) If v $\geq$ 0 and a > 0, the speed is increasing.
ii) If v $\geq$ 0 and a < 0, the speed is decreasing.
iii) If v $\leq$ 0 and a > 0, the speed is decreasing.
iv) If v $\leq$ 0 and a < 0, the speed is increasing.

ACCELERATION:

$$a = \frac{dv}{dt} = \frac{dv}{ds} \bullet \frac{ds}{dt} = v \bullet \frac{dv}{ds}$$

$$a = \frac{d^2s}{dt^2}$$

(Note: speed = $|v(t)|$)

DISTANCE: If $v = f(t)$, the distance traveled by a body between $t = a$ and $t = b$ is given by

$$\int_a^b |f(t)|\, dt$$

(Be careful. Does the object change directions between a and b?)

EQUATION OF A TANGENT LINE:

$$y - y_1 = f'(x_1) \bullet (x - x_1)$$

EQUATION OF A NORMAL LINE:

$$y - y_1 = \left[-\frac{1}{f'(x_1)} \right] \bullet (x - x_1)$$

TANGENTS (function must exist at x_i)

<u>Vertical tangents</u>: $f'(x_i)$ does not exist

<u>Horizontal tangents</u>: $f'(x_i) = 0$

LINEAR APPROXIMATION The linear approximation to $f(x)$ near $x = x_o$ is given by $y = y_o + f'(x_o) \cdot (x - x_o)$ for x sufficiently close to x_o.

EULER'S METHOD ("Numerical Solutions to a Differential Equation")

Iterative use of the Linear Approximation with a given step value.

$$y_1 = y_0 + f'(x_o) \cdot (x_1 - x_o)$$

$$y_2 = y_1 + f'(x_1) \cdot (x_2 - x_1)$$

$$y_3 = y_2 + f'(x_2) \cdot (x_3 - x_2)$$

etc.

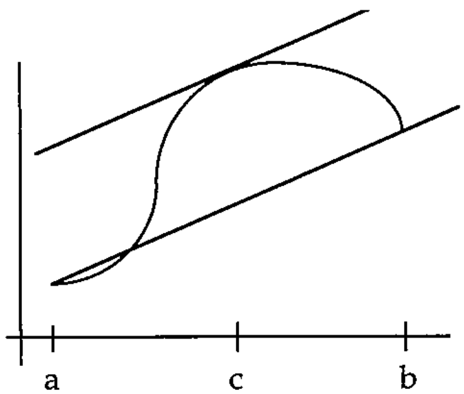
INVERSES: To find the inverse of y = f(x), solve for x in terms of y, then *interchange* x and y.

$$f[f^{-1}(x)]=x \qquad \text{and} \qquad f^{-1}[f(x)]=x$$

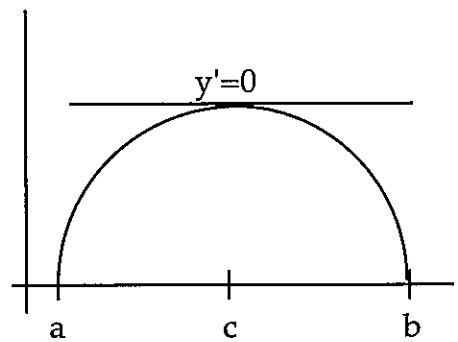
$$\left(f^{-1}\right)'(d)=\frac{1}{f\ (c)} \quad or \quad \frac{dx}{dy}=\frac{1}{dy/dx}$$

MEAN-VALUE THEOREM (SPECIAL CASE -- ROLLE'S THEOREM): If the function f(x) is continuous at each point on the closed interval a $\leq x \leq$ b and has a derivative at each point on the open interval a < x < b, then there is at least one number c, a < c < b, such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$



MEAN VALUE THEOREM



ROLLE'S THEOREM

"Where average velocity $\left(\frac{f(b)-f(a)}{b-a}\right)$ *meets instantaneous velocity* $(f'(c))$*."*

ABSOLUTE-VALUE THEOREM:

$$f(x)=|x|=\begin{cases} x, & if \quad x \geq 0 \\ -x, & if \quad x < 0 \end{cases}$$

GREATEST-INTEGER THEOREM:

g(x) = [x] is the greatest integer <u>not greater</u> than x.

e.g. g(5.2) = 5, g(-1.5) = -2, g(1) = 1

DIRECT VARIATION: $y=kx$ ("y" is directly proportional to "x")

INVERSE VARIATION: $y=\frac{k}{x}$ *or* $xy=k$ ("y" is inversely proportional to "x")

REFLECTIONS:

The graph of $y = -f(x)$ is the reflection of $y = f(x)$ in the x-axis;

eg. $y = x^2$; $y = -x^2$

whereas the graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ in the y-axis.

eg. $y = \sqrt{x}$; $y = \sqrt{-x}$

ODD/EVEN FUNCTIONS:

EVEN: $f(-x) = f(x)$

ODD: $f(-x) = -f(x)$

e. g. Even function: $y = x^2$ *or* $y = \cos x$

Odd function: $y = x^3$ *or* $y = \sin x$

SYMMETRY:

w.r.t. x-axis	....	equivalent equations when y replaced by -y
w.r.t. y-axis	....	equivalent equations when x replaced by -x
w.r.t. origin	....	equivalent equations when x replaced by -x *and* y replaced by -y

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RELATIONSHIPS between the graphs of and the graphs of $y = f(x)$ and the graphs of $y = kf(x)$, $y = f(kx)$, $y - k = f(x - h)$, $y = |f(x)|$, *and* $y = f(|x|)$.

LOGARITHMIC FUNCTIONS:

$y = \log_a x$ *iff* $a^y = x$

$y = \ln x$ iff $e^y = x$

PROPERTIES:

$$\begin{cases} \ln(ab) = \ln a + \ln b \\ \ln\left(\frac{a}{b}\right) = \ln a - \ln b \\ \ln a^r = r \bullet \ln a \\ \log_a x = \frac{\ln x}{\ln a} \end{cases}$$

$\ln 1 = 0$ $\quad x^a \cdot x^b = x^{a+b}$ $\quad x^{a/b} = \sqrt[b]{x^a}$

$\ln e = 1$ $\quad \frac{x^a}{x^b} = x^{a-b}$ $\quad x^o = 1$

$\ln e^x = x$

$e^{\ln x} = x$ $\quad (x^a)^b = x^{ab}$ $\quad x^{-a} = \frac{1}{x^a}$

NATURAL LOGARITHM: $\ln x = \int_1^x \frac{dt}{t}$

EQUATIONS FOR EXPONENTIAL GROWTH AND DECAY: Equations of the form $y'=ky$ are solved as.

$$A= A_o e^{kt} \qquad or \qquad A=A_o e^{rt}$$

LAWS OF LOGISTIC GROWTH : Equations of the form $y' = ky(A - y)$.

$$y = \frac{A}{1+Be^{-kt}}$$

NB. A=the Maximum Capacity and the POI $\left(x, \frac{A}{2}\right)$ is the moment of maximum growth.

SLOPE FIELDS

Tips associating the slope field to a particular Differential Equation:

1) Horizantal Dashes $\rightarrow \frac{dy}{dx} = 0$

2) Dashes \ $\rightarrow \frac{dy}{dx} < 0$

3) Dashes / $\rightarrow \frac{dy}{dx} > 0$

4) All Dashes in any column // to each other $\rightarrow \frac{dy}{dx}$ has no y

5) All Dashes in any row // to each other $\rightarrow \frac{dy}{dx}$ has no x

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INTEGRATION FORMULAS:

$\int f(x)\,dx = F(x) + C, \text{ where } F'(x) = f(x)$

$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ [First Fundamental Theorem]

Remember the Chain Rule!!!: $\frac{d}{dx}\int_a^u f(t)\,dt = f(u)\cdot D_u$

$\int_a^b f(x)\,dx = F(b) - F(a), \text{ where } F'(x) = f(x)$ [Second Fundamental Theorem]

$\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$

$\int \frac{du}{u} = \ln|\ u\ | + C$

$\int e^u\,du = e^u + C$

$\int a^u\,du = \frac{a^u}{\ln a} + C, (a > 0,\ a \neq 1)$

$\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \sec^2 u\,du = \tan u + C$

$\int \csc^2 u\,du = -\cot u + C$

$\int \sec u \bullet \tan u\,du = \sec u + C$

$\int \csc u \bullet \cot u\,du = -\csc u + C$

$\int \sec u\,du = \ln|\ \sec u + \tan u\ | + C$

$\int \tan u\,du = \ln|\ \sec u\ | + C$

$\int \csc u\,du = \ln|\ \csc u - \cot u\ | + C$

$\int \cot u\,du = \ln|\ \sin u\ | + C$

$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin\ 2u + C$

$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin\ 2u + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$

$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \bullet \tan^{-1}\left(\frac{u}{a}\right) + C$

$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \bullet \sec^{-1}\left|\ \frac{u}{a}\ \right| + C$

$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right) + C$

Integration by Parts: $\int u\,dv = uv - \int v\ du$

Integral Boundary Rules

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\text{If} \quad f(x) \le g(x) \quad \text{on} \quad [a,b], \quad \text{then} \quad \int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$

AVERAGE (MEAN) VALUE: If the function $y = f(x)$ is continuous on the interval $a \le x \le b$, then the average or mean value of y with respect to x over the interval [a,b] is

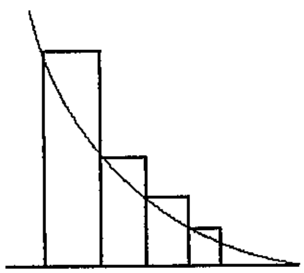
$$(y_{av})_x = \frac{1}{b-a}\int_a^b f(x)dx$$

AREA APPROXIMATIONS

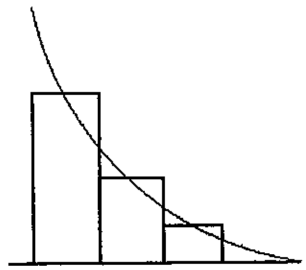
RIEMANN SUMS

$$A = \lim_{n \to +\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

$$A = \int_a^b f(x)\,dx$$



Left-Hand Rectangles



Midpoint Rectangles

Right-Hand Rectangles

TRAPEZOIDAL RULE:

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+....+2f(x_{n-1})+ \quad f(x_n)\right]$$

AREA FORMULAS

Function: $A = \int_a^b [f(x)-g(x) \quad]dx \quad or\ A = \int_c^d [f(y)-g(y) \quad]dy$

Polar: $A = \frac{1}{2}\int_\alpha^\beta [r(\theta)]^2\, d\theta$

Parametric: Eliminate the parameter:
i) isolate the parameter in one equation, and
ii) substitute into the other equation
and then use the Function formula

VOLUME

SOLIDS WITH KNOWN CROSS SECTIONS (SLICING)

$$V = \int_a^b A(x)dx \quad or \quad V = \int_c^d A(y)dy$$

CIRCULAR DISK METHOD (rectangles perpendicular and attached)

$$V = \pi \int_a^b [R(x)]^2 dx \quad or \quad V = \pi \int_c^d [R(y)]^2 dy$$

CIRCULAR RING/WASHER METHOD (rectangles perpendicular and not attached)

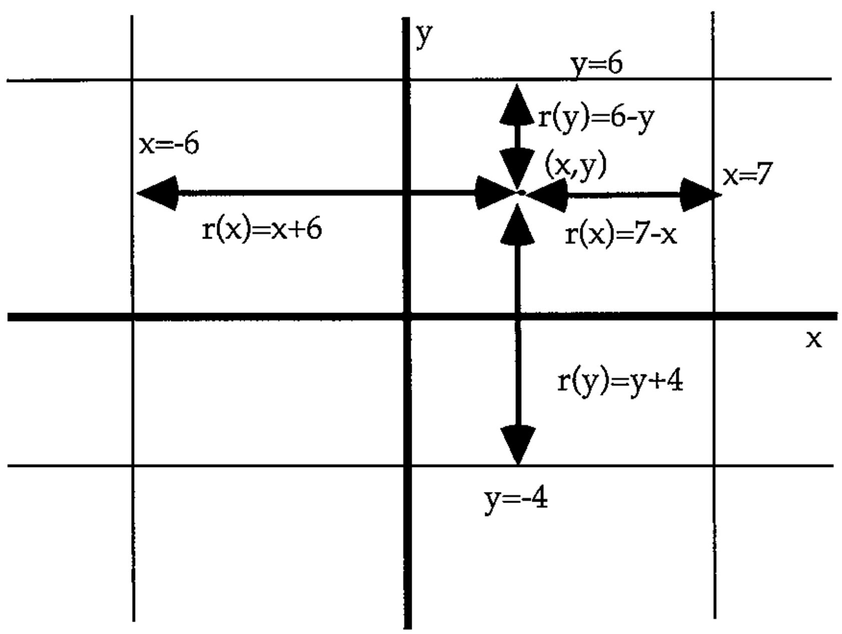
$$V = \pi \int_a^b \{[R(x)]^2 - [r(x)]^2\} dx \quad or \quad V = \pi \int_c^d \{[R(y)]^2 - [r(y)]^2\} dy$$

CYLINDRICAL SHELL METHOD (rectangles parallel)

$$V = 2\pi \int_a^b r(x) \bullet h(x)dx \quad or \quad V = 2\pi \int_c^d r(y) \bullet h(y)dy$$

where r(x) or r(y) is the distance from the curve to the axis around which it is rotating

VOLUME REFERENCE:



ARC LENGTH:

Function: $$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Polar: $$L=\int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Parametric: $$L=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

PARAMETRIC, POLAR AND VECTOR FORMS

Parametric: $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ (a function in t) and $\frac{d^2y}{dx^2}=\frac{dy'/dt}{dx/dt}$

Vertical Tangent: $\frac{dx}{dt}=0$

Horizontal Tangent: $\frac{dy}{dt}=0$

Area: Eliminate the parameter and use the Function formula

Arc length: $L=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

Polar: Area: $A=\frac{1}{2}\int_\alpha^\beta [r(\theta)]^2\,d\theta$

Arc Length: $L=\int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

Parametric Polar: $x(\theta)=r\cdot\cos\theta$ and $y(\theta)=r\cdot\sin\theta$

Vector: Velocity $\vec{v}=x'(t)\,\vec{i}+y'(t)\,\vec{j}$

Speed $=\left|\vec{v}\right|=\sqrt{[x'(t)]^2+[y'(t)]^2}$

TRIG IDENTITIES

$$\sin\theta = \frac{1}{\csc\theta} \qquad \sin^2\theta + \cos^2\theta = 1 \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta} \qquad \cot^2\theta + 1 = \csc^2\theta \qquad = 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

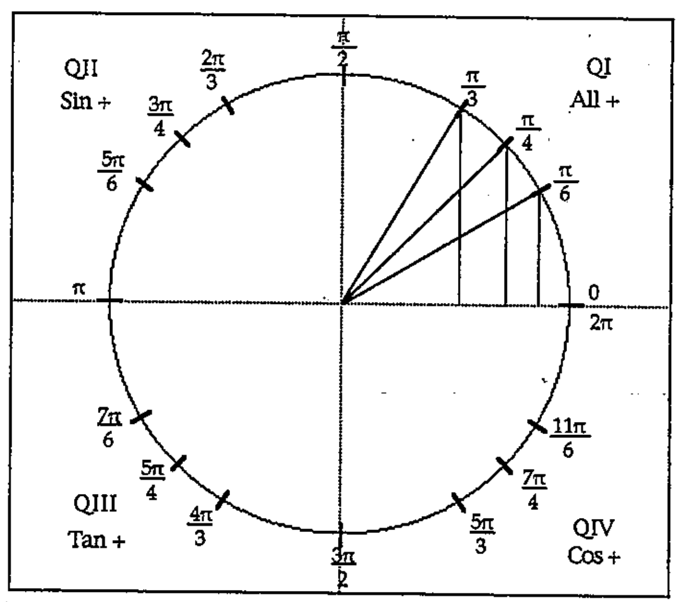
$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\cos^{-1}x = \left\{\begin{matrix} \text{calculator} \pm 2\pi n \\ -\text{calculator} \pm 2\pi n \end{matrix}\right\} \qquad \sin^{-1}x = \left\{\begin{matrix} \text{calculator} \pm 2\pi n \\ \pi - \text{calculator} \pm 2\pi n \end{matrix}\right\}$$

$$\tan^{-1}x = \left\{\begin{matrix} \text{calculator} \pm 2\pi n \\ \pi + \text{calculator} \pm 2\pi n \end{matrix}\right\} = \text{calculator} \pm \pi n$$

The Table

Rads	Deg	Cos x	Sin x	Tan x
0	0	1	0	0
$\frac{\pi}{6}$	30	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	60	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90	0	1	∞
$\frac{2\pi}{3}$	120	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	135	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1
$\frac{5\pi}{6}$	150	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$
π	180	-1	0	0
$\frac{7\pi}{6}$	210	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{5\pi}{4}$	225	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
$\frac{4\pi}{3}$	240	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{3\pi}{2}$	270	0	-1	∞
$\frac{5\pi}{3}$	300	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$
$\frac{7\pi}{4}$	315	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
$\frac{11\pi}{6}$	330	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$
2π	360	1	0	0

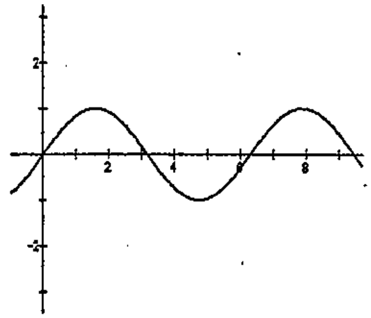


GRAPHING TRIG FUNCTIONS:

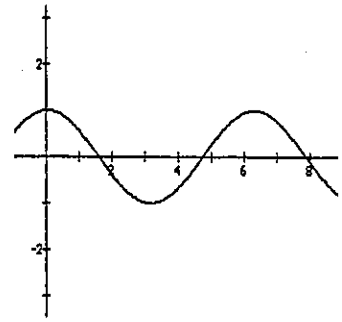
$y = C + A\cos B(x - D)$; $p = \dfrac{2\pi^R}{B}$ or $p = \dfrac{360^\circ}{B}$

$y = C + A\tan B(x - D)$; $p = \dfrac{\pi^R}{B}$ or $p = \dfrac{180^\circ}{B}$

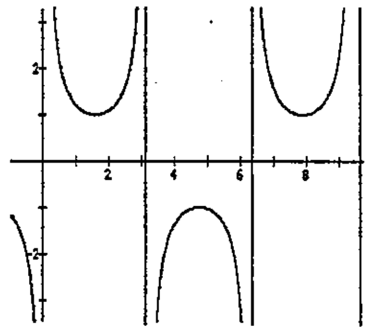
A) Amplitude
B) Period
C) Vertical Shift
D) Horizontal Shift



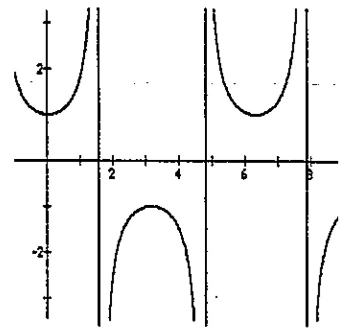
y = sin x



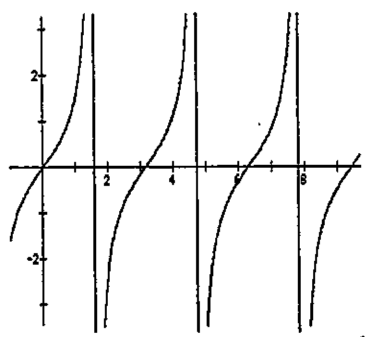
y = cos x



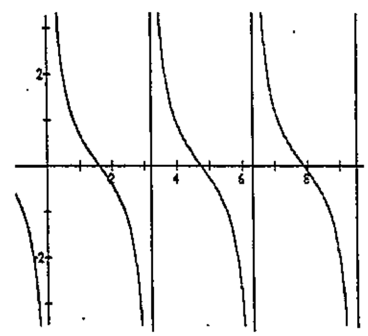
y = csc x



y = sec x



y = tan x

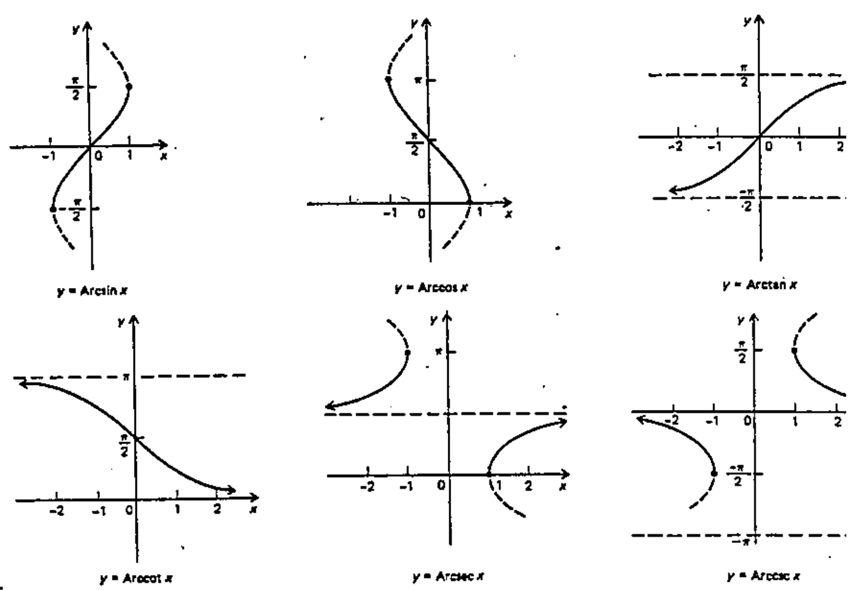


y = cot x

TRIG FUNCTIONS AND THEIR INVERSES:

$y = \sin x$	domain: $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ range: $-1 \le y \le 1$	$y = \sin^{-1}x$	domain: $-1 \le x \le 1$ range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos x$	domain: $0 \le x \le \pi$ range: $-1 \le y \le 1$	$y = \cos^{-1}x$	domain: $-1 \le x \le 1$ range: $0 \le y \le \pi$
$y = \tan x$	domain: $-\frac{\pi}{2} < x < \frac{\pi}{2}$ range: $y \in \{Reals\}$	$y = \tan^{-1}x$	domain: $x \in \{Reals\}$ range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

1) Domain
2) Axis Points
3) Phase Shift
4) Vertical Asymptotes
5) Extremes
6) Range



Inverse circular functions

HORIZONTAL ASYMPTOTES (Maximum Capacity)

$$\lim_{x\to\infty} f(x) \quad \text{or} \quad \lim_{x\to-\infty} f(x)$$

$y = \text{Tan}^{-1}\ x$ $\qquad \lim_{x\to\infty} f(x) = \frac{\pi}{2}$ or $\lim_{x\to-\infty} f(x) = -\frac{\pi}{2}$

$y = e^x$ $\qquad \lim_{x\to\infty} f(x) = \infty$ or $\lim_{x\to-\infty} f(x) = 0$

$y' = ky(A - y)$ $\qquad \lim_{x\to\infty} f(x) = A$

L'Hôpital's Rule: If $\lim_{x\to x_0}\left[\frac{f(x)}{g(x)}\right]$ is indeterminate of the form $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$, and if $\lim_{x\to x_0}\left[\frac{f'(x)}{g'(x)}\right]$ exists, then $\lim_{x\to x_0}\left[\frac{f(x)}{g(x)}\right] = \lim_{x\to x_0}\left[\frac{f'(x)}{g'(x)}\right]$.

IMPROPER INTEGRALS

1. Boundary at infinity: $\int_a^{\infty} f(x)dx = \lim_{b\to\infty}[F(b) - F(a)]$

2. Boundary is a Veritical Asymptote: $\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx$

   or $= \lim_{c\to a^+}\int_c^b f(x)dx$

3. Region includes a Vertical Asymptote at x=c: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

TAYLOR POLYNOMIALS

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^n(a)(x-a)^n}{n!} + R_n(x)$$

where $R_n(x) = \dfrac{f^{n+1}(c)(x-c)^{n+1}}{(n+1)!}$ for some $c \in (x,\ a)$

McLauren series=Taylor Series where a=0

SERIES OF KNOWN FUNCTIONS

$$y = \sin\ x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)\ !} + \cdots = \sum_0^\infty \frac{(-1)^n x^{2n+1}}{(2n+1)\ !}$$

$$**y = \cos\ x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)\ !} + \cdots = \sum_0^\infty \frac{(-1)^n x^{2n}}{(2n)\ !}$$

$$y = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_0^\infty \frac{x^n}{n!}$$

$$y = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_0^\infty x^n \quad \text{on } -1 < x < 1$$

$$**y = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-x)^n + \cdots = \sum_0^\infty (-1)^n x^n \quad \text{on } -1 < x < 1$$

$$**y = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + (-x)^{2n} + \cdots = \sum_0^\infty (-1)^n x^{2n} \quad \text{on } -1 < x < 1$$

$$**y = \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots = \sum_0^\infty \frac{(-1)^n x^{2n+1}}{2n+1} \quad \text{on } -1 \le x \le 1$$

$$**y = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^n x^n}{n} + \cdots = \sum_1^\infty \frac{(-1)^n x^n}{n} \quad \text{on } -1 < x \le 1$$

**These can be derived from the unmarked series.

If $\sum_{1}^{\infty} a_n$ IS an Alternating Series:

Alternating Series Definition: $\sum_{1}^{\infty} (-1)^{n+1} a_n$ or $\sum_{0}^{\infty} a_n \cos \pi n$

Liebnitz Alternating Series Test: $\sum_{1}^{\infty} (-1)^{n+1} a_n$ converges if

1. all a_n are positive,

2. $a_n \geq a_{n+1}$,

and 3. $\lim_{n \to \infty} a_n = 0$

Absolute Convergence vs Conditional Convergence (only applies to Alternating Series)

$\sum a_n$ is absolutely convergent if $\sum | a_n |$ converges.

$\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum | a_n |$ diverges.

RADIUS OF CONVERGENCE

R is the radius of convergence when $\lim_{n \to \infty} \left| \frac{a_{n+1}(x-a)^{n+1}}{a_n (x-a)^n} \right| < 1$ leads to $| x - a | < R$

INTERVAL OF CONVERGENCE

Solve $| x - a | < R$ (from the Radius of Convergence) and test convergence at the endpoints

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SPECIAL LIMITS (for comparison)

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\lim_{n \to \infty} \frac{\ln n}{n} = 0$

$\lim_{n \to \infty} \sqrt[n]{n} = 1$

$\lim_{n \to \infty} x^{1/n} = 1$

$\lim_{n \to \infty} x^n = 0$, if $| x | < 1$

$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x$

$\lim_{n \to \infty} \frac{x^n}{n!} = 0$

$\lim_{n \to \infty} \frac{n!}{x^n} = \infty$

CONVERGENCE/DIVERGENCE

TESTS: Divergence Test (nth term test): If $\lim\limits_{n\to\infty} a_n \neq 0 \to$ it diverges.

If $\lim\limits_{n\to\infty} a_n = 0 \to$ no conclusion

Cauchy Ratio Test: If $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1 \to$ it converges absolutely

If $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1 \to$ it diverges

If $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1 \to$ no conclusion

If $\sum_1^\infty a_n$ is NOT an Alternating Series:

Integral Test: If f(x) is a decresing function, then $\sum_1^\infty a_n$ and $\int_1^\infty x_n dx$ either both converge or both diverge.

Comparison Test: If $\sum_1^\infty b_n$ converges and $a_n \leq b_n$, then $\sum_1^\infty a_n$ converges.

If $\sum_1^\infty b_n$ diverges and $a_n \geq b_n$, then $\sum_1^\infty a_n$ diverges.

Limit Comparison Test

1. If $\lim\limits_{n\to\infty} \left|\frac{a_n}{b_n}\right| > 0 \to$ then both converge or both diverge.

2. If $\lim\limits_{n\to\infty} \left|\frac{a_n}{b_n}\right| = 0$ and $\sum b_n$ converges $\to$ then $\sum a_n$ converges.

3. If $\lim\limits_{n\to\infty} \left|\frac{a_n}{b_n}\right| = \infty$ and $\sum b_n$ diverges $\to$ then $\sum a_n$ diverges.

The nth Root Test: Suppose $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = p$. Then,

1. if p<1, $\sum a_n$ converges
2. if p>1, $\sum a_n$ diverges
3. if p=1, no conclusion

KNOWN SERIES (for comparison)

Geometric Series: $\sum_{1}^{\infty} ar^{n}$ --converges to $\frac{a}{1-r}$ for r<1

--diverges for r≥1

p-series: $\sum_{1}^{\infty} \frac{1}{n^{p}}$ --converges for p>1

--diverges for p≤1

Harmonic Series: $\sum_{1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ Diverges

Alternating Harmonic Series: $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ Converges conditionally

Telescoping Series: Any series that can be simplified by Partial Fractions such that consecutive terms add to 0, leaving only the first and last terms $\left(\text{e.g., } \sum_{1}^{\infty} \frac{1}{n(n+1)} \right)$ It will generally converge, by the integral Test and partial fractions.

<u>AP CALCULUS BC VOCABULARY</u>

absolute convergence
absolute minimum
absolute maximum
acceleration
acceleration vector
algebraic function
alternating series
amplitude
antiderivative
antidifferentiation
arc length
arccosine
arcsine
arctangent
asymptote
average rate of change
axis of rotation
axis of symmetry
base (exponential and log)
bounded above
bounded below
bounded
cartioid
Cartesian Coordinate System
Chain Rule
circle
circular functions
closed interval [a,b]
coefficient
Comparison Test
complex number
components of a vector
composition f ° g
concave down
concave up
conditional convergence
conic section
constant function
constant of integration
continuity at a point
continuity on an interval
continuous function
convergent improper integral
convergent sequence
convergent series
coordinate axes
cosecant function
cosine function
cotangent function
critical point
critical value
cross-sectional area
decay model
decreasing function
decreasing on an interval
definite integral
degree
delta notation
derivative
difference quotient
differentiability
differential
differential equation
differentiation
discontinuity
disk method
distance (from velocity)
distance formula
divergent improper integral
divergent sequence
divergent series
domain
dummy variable of integration
dy/dx (leitniz notation)
e
ellipse
end behavior
endpoint extrema
essential discontinuity
Euler's Method
even function
exponential function
exponential growth and decay
exponential laws
extremum
factorial

AP CALCULUS BC VOCABULARY

First Derivative Test
Frequency of a periodic function
function
Fundemental Theorem of Calculus
geometric sequence
geometric series
graph
growth models
growth rate
half-life
harmonic series
hyperbola
imaginary number
implicit differentiation
improper integral
increasing function
increasing on an interval
increment
indefinite integral
indterminate form
infinite limit
inflection point
initial condition
initial value problem
inscribed rectangle
instantaneous rate of change
instantaneous velocity
integer
integrable function
integrand
integration by partial fractions
integration by parts
integration by substitution
Intermediate Value Theorem
interval
interval of convergence
inverse function
irrational number
Lagrange Error Bound
Law of Cosines
Law of Sines
left-hand limit
left-hand sum
Leibniz, Gottfried
L'Hopital's Rule
limit
limit at infinity
limit of integration
linear approximation
linear function
local extrema
local linearity
local linearization
logarithmic function
logarithmic laws
logistic equation
logistic growth
lower bound
Maclaurin series
maximum
mean value
Mean Value Theorem
midpopint formula
minimum
monotonic
motion
natural log
Newton, Isaac
non-removable discontinuity
normal line
numerical derivative
numerical integration
odd function
one-to-one function
open interval (a,b)
optimization
order of a derivative
origin
parabola
parallel curves
parameter
parametric curve
partial fractions
partial sum of a series
partition of an interval
percentage error

AP CALCULUS BC VOCABULARY

period
periodic function
perpendicular curves
piece-wise defined functions
polar coordinates
polynomial
position function
position vector
power series
prime notation f'(x)
Product Rule
proportionality
p-series
quadrant
quadratic formula
Quotient Rule
radian
radius of a circle
radius of convergence
range
rate of change
rational function
Ratio Test
real number
rectangular coordinates
region (in a plane)
related rates
relative error
relative maximum
relative minimum
removable discontinuity
Rhiemann sum
right-hand limit
right-hand sum
root of an equation
roundoff error
scalar
secant function
secant line
second derivative
Second Derivative Test
separable differential equation
sequence
series
set
sigma notation
sine function
slope
slope field
solid (in 3-space)
solid of revolution
speed
sphere
subset
symmetry
tangent function
tangent line
tangent vector
Taylor polynomial
Taylor series
term of a sequence or series
transcendental function
Trapezoidal Rule
truncation error for power series
trigonometric functions
unit circle
unit vector
upper bound
u-substitution
vector
vertex
viewing window
volume by slicing
x-axis
x-intercept
y-axis
y-intercept
zero of a function