Calculus '21-22
Anti-Derivative Practice Test

Name Solution Key

Score___________

1. $\int\left(5x^3+5^x-\frac{1}{\sqrt[5]{x^3}}+\frac{1}{5x^3}\right)dx$

$= \int\left(5x^3+5^x-x^{-3/5}+\frac{1}{5}x^{-3}\right)dx$

$= \frac{5x^4}{4}+\frac{5^x}{\ln 5}-\frac{x^{2/5}}{2/5}+\frac{1}{5}\frac{x^{-2}}{-2}+C$

$= \frac{5}{4}x^4+\frac{5^x}{\ln 5}-\frac{5}{2}x^{2/5}-\frac{1}{10}x^{-2}+C$

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2. $\int\frac{\sec(\ln x)}{x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$= \int \sec u\, du$

$= \ln|\sec u+\tan u|+C$

$= \ln|\sec(\ln x)+\tan(\ln x)|+C$

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3. $\int \left(x^5 - x^3 \csc^2\left(2x^4\right) + xe^{x^2}\right) dx$

$\int x^5 dx - \frac{1}{8}\int \csc^2 2x^4 \, 8x^3 dx \quad + \frac{1}{2}\int e^{x^2}(2x\, dx$

$u = 2x^4 \Rightarrow du = 8x^3$

$= \frac{x^6}{6} - \frac{1}{8}\int \csc^2 u_1 du \qquad + \frac{1}{2}\int e^{u_2} du$

$= \frac{1}{6}x^6 - \frac{1}{8}(-\cot u_1) + \frac{1}{2}e^{u_2} + C$

$= \frac{1}{6}x^6 + \frac{1}{8}\cot(2x^4) + \frac{1}{2}e^{x^2} + C$

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4. $\int \left(\frac{e^x}{\tan e^x}\right) dx =$

$u = e^x$

$du = e^x dx$

$= \int \cot u \, du$

$= \ln|\sin u| + C$

$= \ln|\sin e^x| + C$

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5. A particle moves along the x-axis with acceleration at any time t given as $a(t)=t^2-\frac{1}{\sqrt{t}}$. If the particle's velocity is 2 and its initial position is -4 when $t=0$, what is the position function?

$$V = \int (t^2 - t^{-1/2})\,dt$$

$$= \frac{1}{3}t^3 + 2t^{1/2} + C_1 \qquad (0,2) \Rightarrow 2 = 0 + 0 + C_1 \Rightarrow C_1 = 2$$

$$= \frac{1}{3}t^3 + 2t^{1/2} + 2$$

$$x(t) = \int \left(\frac{1}{3}t^3 + 2t^{1/2} + 2\right) dt$$

$$= \frac{1}{3}\frac{t^4}{4} + \frac{2t^{3/2}}{3/2} + 2t + C_2$$

$$(0,-4) \Rightarrow -4 = 0 + 0 + 0 + C_2 \Rightarrow C_2 = -4$$

$$x(t) = \frac{1}{12}t^4 + \frac{4}{3}t^{3/2} + 2t - 4$$

6. $\int\left(\frac{1}{x\sqrt{x^2-16}}-\frac{1}{16+x^2}+\frac{x}{16+x^2}\right)dx$

$= \int \frac{1}{x\sqrt{x^2-16}}dx - \int \frac{1}{16+x^2} dx + \int \frac{x}{16+x^2} dx$

$u=x \quad a=4$ $\qquad$ $u=x$, $a=4$ $\qquad$ $u=16+x^2$, $du=2x\,dx$

$= \frac{1}{4}\sec^{-1}\frac{x}{4} - \frac{1}{4}\tan^{-1}\frac{x}{4} + \frac{1}{2}\int u^{-1}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{4}\sec^{-1}\frac{x}{4} - \frac{1}{4}\tan^{-1}\frac{x}{4} + \frac{1}{2}\ln|16+x^2| + C$

7. The velocity of an object in rectilinear motion is given by $v(t)=t^5-16t^3$ with $x(0)=2$.

a) Find the position equation of the object. What is the position at $t=1$?

$$x(t)=\int v\,dt=\int (t^5-16t^3)\,dt$$

$$x=\frac{1}{6}t^6-4t^4+C$$

$$(0,2)\rightarrow 2=0+0+C \Rightarrow C=2$$

$$x(t)=\frac{1}{6}t^6-4t^4+2$$

$$x(1)=\frac{1}{6}-4-2=-\frac{35}{6}$$

b) Find the acceleration equation of the object. What is the acceleration at $t=1$?

$$a(t)=v'(t)=5t^4-48t^2$$

$$a(1)=5-48=-43$$

8. Find the particular solution $w = f(t)$ that passes through ~~(1, 2)~~ $(0, 3)$ if

$$\frac{dw}{dt} = w(6 - 2t)$$

$$\int \frac{1}{w}\,dw = \int (6 - 2t)\,dt$$

$$e^{\ln|w|} = e^{6t - \frac{2t^2}{2} + C}$$

$$|w| = e^{6t - t^2 + c} = Ke^{6t - t^2}$$

$$(0, 3) \rightarrow 3 = Ke^{0} = K$$

$$w = 3e^{6t - t^2}$$