Calculus '20-21
Anti-Derivative Test

Name Solution Key

Score____________

1. $\int \left(2x^3 + 2^x - \frac{1}{\sqrt[3]{x^2}} + \frac{1}{3x^2}\right) dx$

$= \int 2x^3\,dx + \int 2^x\,dx - \int x^{-2/3}\,dx + \int \frac{1}{3} x^{-2}\,dx$

$= \frac{1}{2}x^4 + \frac{2^x}{\ln 2} - \frac{x^{1/3}}{1/3} + \frac{1}{3}\cdot\frac{x^{-1}}{-1} + C$

$= \frac{1}{2}x^4 + \frac{2^x}{\ln 2} - 3x^{1/3} - \frac{1}{3}x^{-1} + C$

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2. $\int \left(\frac{\csc^2\sqrt{x}}{\sqrt{x}}\right) dx$

$u = x^{1/2}$
$du = \frac{1}{2}x^{-1/2}$

$= -2\int \csc^2 x^{1/2} \left(\frac{1}{2x^{1/2}}\,dx\right)$

$= -2\int \csc^2 u\,du$

$= -2(-\cot u) + C$

$= 2\cot\sqrt{x} + C$

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3. $\int\left(3\sqrt{x^3}-\sec(2x)+\frac{x}{e^{4x^2}}\right)dx$

$= \int 3x^{3/2} - \frac{1}{2}\int \sec 2x\,(2dx) + \left(\frac{-1}{8}\right)\int e^{-4x^2}(-8x\,dx)$

$= \int 3x^{3/2}dx - \frac{1}{2}\int \sec u\,du - \frac{1}{8}\int e^{u}du$

$= 3\frac{x^{5/2}}{5/2} - \frac{1}{2}\ln|\sec u + \tan u| - \frac{1}{8}e^{u} + C$

$= \frac{6}{5}x^{5/2} - \frac{1}{2}\ln|\sec 2x + \tan 2x| - \frac{1}{8}e^{-4x^2} + C$

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4. $\int\left(\frac{7w^2-4w+1}{5w^3}\right)dw = \int\left(\frac{7}{5}w^{-1} - \frac{4}{5}w^{-2} + \frac{1}{5}w^{-3}\right)dw$

$= \frac{7}{5}\ln|w| - \frac{4}{5}\frac{w^{-1}}{-1} + \frac{1}{5}\frac{w^{-2}}{-2} + C$

$= \frac{7}{5}\ln|w| + \frac{4}{5}w^{-1} - \frac{1}{10}w^{-2} + C$

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5. The acceleration of a particle is described by $a(t)=e^{3t}-3t^2$. Find the distance equation for $x(t)$ if $v(0)=\frac{6}{5}$ and $x(0)=1$.

$u=3t$
$du=3dt$

$$V=\int(e^{3t}-3t^2)\,dt=\int e^{3t}\,dt-\int 3t^2\,dt$$

$$V=\frac{1}{3}e^{3t}-t^3+C_1$$

$$\frac{6}{5}=\frac{1}{3}e^0-0+C_1 \rightarrow C_1=\frac{13}{15}$$

$$x(t)=\int\left(\frac{1}{3}e^{3t}-t^3+\frac{13}{15}\right)dt$$

$$=\frac{1}{9}e^{3t}-\frac{1}{4}t^4+\frac{13}{15}t+C_2$$

$$1=\frac{1}{9}-0+0+C_2 \rightarrow C_2=8/9$$

$$x(t)=\frac{1}{9}e^{3t}-\frac{1}{4}t^4+\frac{13}{15}t+\frac{8}{9}$$

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6. $\displaystyle\int\left(\frac{1}{x\sqrt{x^2-9}}+\frac{1}{\sqrt{9-x^2}}+\frac{x}{\sqrt{9-x^2}}\right)dx$

$$\int\frac{1}{x\sqrt{x^2-9}}\,dx+\int\frac{1}{\sqrt{9-x^2}}\,dx+\left(-\frac{1}{2}\right)\int\left(\frac{-2x}{\sqrt{9-x^2}}\,dx\right)$$

$$\frac{1}{3}\sec^{-1}\frac{x}{3}+\sin^{-1}\left(\frac{x}{3}\right)+\left(-\frac{1}{2}\right)\int u^{-1/2}\,du$$

$$\text{''}\quad+\quad\text{''}\quad-\frac{\cancel{1}}{\cancel{2}}\,\frac{u^{1/2}}{\cancel{1/2}}$$

$$\frac{1}{3}\sec^{-1}\frac{x}{3}+\sin^{-1}\frac{x}{3}-(9-x^2)^{1/2}+C$$

7. The velocity of an object in rectilinear motion is given by $v(t) = t^5 - 16t^3$ with $x(0) = 2$.

a) Find the position equation of the object. What is the position at $t = 1$?

$$x(t) = \int v\,dt = \int (t^5 - 16t^3)\,dt$$

$$= \frac{1}{6}t^6 - 4t^4 + c$$

$$x(0) = 2 \rightarrow 2 = 0 - 0 + c \Rightarrow c = 2$$

$$x(t) = \frac{1}{6}t^6 - 4t^4 + 2$$

$$x(1) = \frac{1}{6} - 4 + 2 = -\frac{11}{6}$$

b) Find the acceleration equation of the object. What is the acceleration at $t = 1$?

$$a(t) = v'(t) = 5t^4 - 48t^2$$

$$a(1) = 5 - 48 = -43$$

8. Find the particular solution to $\frac{dy}{dx} = \frac{x+1}{y}$ at $(-1, 2)$.

$$\int y\,dy = \int (x+1)\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

$$(-1,2) \rightarrow \frac{2^2}{2} = \frac{(-1)^2}{2} + (-1) + C$$

$$\frac{5}{2} = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + \frac{5}{2}$$

$$y^2 = x^2 + 2x + 5$$

$$y = \pm\sqrt{x^2+2x+5}$$

BUT $(-1,2) \rightarrow$ $y = +\sqrt{x^2+2x+5}$