Calculus '21-22 Name Solutions Key
Integral Practice Test

Score____________

1. Evaluate $\int_2^3 \left(x^2+4x+3\right) dx$ Show the antiderivative step.

$$= \frac{x^3}{3} + 2x^2 + 3x\Big]_2^3$$

$$= \left(9 + 18 + 9\right) - \left(\frac{8}{3} + 8 + 6\right)$$

$$= 19\frac{1}{3}$$

2. Evaluate $\int_{e^{\frac{\pi}{4}}}^{e^{\frac{\pi}{2}}} \frac{\csc^2(\ln y)}{y} dy$. Show the u-sub and antiderivative.

$u = \ln y$ $\quad u(e^{\pi/2}) = \pi/2$

$du = \frac{1}{y} dy$ $\quad u(e^{\pi/4}) = \pi/4$

$$= \int_{\pi/4}^{\pi/2} \csc^2 u \, du$$

$$= -\cot u\Big]_{\pi/4}^{\pi/2}$$

$$= -\left(\cot \pi/2\right) - \left(-\cot \pi/4\right)$$

$$= 0 + 1 = 1$$

3. Evaluate $\int_0^{\pi/6} \sin(3x)\,dx$. Show the u-sub and antiderivative.

$u = 3x \qquad u(0) = 0$

$du = 3\,dx \qquad u(\pi/6) = \pi/2$

$= \int_0^{\pi/2} \sin u\,du$

$= -\cos u\Big]_0^{\pi/2} = -\cos \pi/2 - (-\cos 0)$

$= 0 + 1$

$= 1$

4. Find the average value of $y = x^2\left(9 - x^3\right)^{1/3}$ on $x \in [1, 2]$. Show the u-sub and antiderivative.

$\frac{1}{2-1}\int_1^2 x^2 (9-x^3)^{1/3}\,dx \qquad u = 9 - x^3$

$du = -3x^2\,dx$

$u(1) = 8$

$u(2) = 1$

$= -\frac{1}{3}\int_8^1 u^{1/3}\,du$

$= -\frac{1}{3}\frac{u^{4/3}}{4/3}\Big]_8^1$

$= -\frac{1}{4}u^{4/3}\Big]_8^1 = \left(-\frac{1}{4}\right) - (-16)$

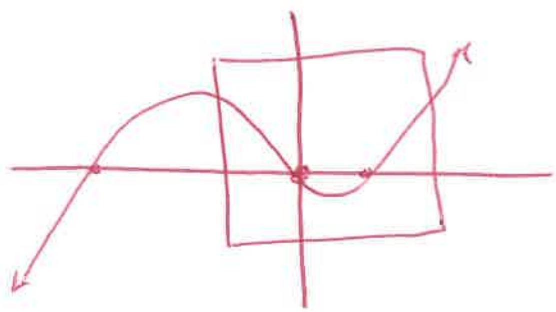
$= 15.75$

5. Consider the function $y = x^3 + 2x^2 - 3x$.

a) Draw the graph of $f(x)$ on $x \in [-1, 2]$.

$x(x^2+2x-3)$

$x(x+3)(x-1)$



b) Prove that a zero of $f(x)$ is at $x = 1$.

$f(1) = 1^3 + 2(1)^2 - 3(1) = 0$

c) Find the exact value of $\int_{-1}^{2} f(x)\,dx$. Show the antiderivative and boundary insertion steps.

$$\int_{-1}^{2}(x^3+2x^2-3x)\,dx$$

$$= \frac{x^4}{4}+\frac{2}{3}x^3-\frac{3}{2}x^2\Big]_{-1}^{2}$$

$$=\left(4+\frac{8}{3}-6\right)-\left(\frac{1}{4}-\frac{2}{3}-\frac{3}{2}\right) = 5.25$$

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d) Find the exact area between the x-axis and $f(x)$ on $x\in[-1,2]$. Show the antiderivative and boundary insertion steps.

$$=\int_{-1}^{0} f(x)\,dx-\int_{0}^{1} f(x)\,dx+\int_{1}^{2} f(x)\,dx$$

$$=\left[\frac{x^4}{4}+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{-1}^{0}-\left[\frac{x^4}{4}+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{0}^{1}+\left[\frac{x^4}{4}+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{1}^{2}$$

$$=\left[0-\left(\frac{1}{4}-\frac{2}{3}-\frac{3}{2}\right)\right]-\left[\frac{1}{4}+\frac{2}{3}-\frac{3}{2}-0\right]+\left[\left(4+\frac{8}{3}-6\right)-\left(\frac{1}{4}+\frac{2}{3}-\frac{3}{2}\right)\right]$$

$$= 6.417$$

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6. Let f be a differentiable function on the closed interval [2, 14] and which has values as shown on the table below.

x	1	6	10	13
$f(x)$	11	18	28	21

a) Using the sub-intervals defined by the table values, use the right-hand Riemann sum to approximate $\int_1^{13} f(x)\,dx$.

$5(18) + 4(28) + 3(21) = 265$

b) Using the sub-intervals defined by the table values, use the trapezoidal sum to approximate $\int_1^{13} f(x)\,dx$

$5\left(\frac{11+18}{2}\right) + 4\left(\frac{18+28}{2}\right) + 3\left(\frac{28+21}{2}\right) = 238$

7. The velocity of a particle moving along the x-axis is given by $v(t)=t^3+t^2-12t$. Find the total distance traveled by the particle between $t=-2$ and $t=5$. Show the set-up and the antiderivative before using the calculator to find the answer.

$t(t^2+t-12)$

$t(t+4)(t-3)$

v		+	0	−	0	+
t	4		0		3	

$\int_{-2}^{5} v(t)\,dt$ = DISPLACEMENT

TOTAL DISTANCE = $\int_{-2}^{0} v(t)\,dt - \int_{0}^{3} v(t)\,dt + \int_{3}^{5} v(t)\,dt$

$= \left[\frac{t^4}{4}+\frac{t^3}{3}-6t^2\right]_{-2}^{0} - \left[\frac{t^4}{4}+\frac{t^3}{3}-6t^2\right]_{0}^{3} + \left[\frac{t^4}{4}+\frac{t^3}{3}-6t^2\right]_{3}^{5}$

$= 882.833$

MISSING TABLE

EC. A small plant is purchased from a nursery and the change in height of the plant is measured at the end of each day for four days. The data, where $H(t)$ is measured in millimeters per day and t is measured in days, are listed above.

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a. Estimate $H'(3)$. Show the work that leads to your answer. Indicate the units.

$$H'(3) = \frac{2.6 - 1.5}{4 - 2} = \frac{1.1}{2} = .55 \text{ mm/DAY/DAY}$$

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b. Considering the units, explain the difference between $H(3)$ and $H'(3)$ in terms of the plant's growth.

H(3) IS THE RATE OF GROWTH OF THE PLANT, IN mm/DAY ON DAY 3

H'(3) = HOW ~~FA~~ FAST THE RATE OF GROWTH IS CHANGING IN MM/DAY/DAY ON DAY 3

t Days	0	1	2	3	4
H(t) IN mm/DAY	0	1.3	1.5	2.1	2.6

c. Use right-hand rectangles with subintervals indicated by the table to approximate $\frac{1}{4}\int_0^4 H(t)\,dt$. Using correct units, explain the meaning of this value in the context of the problem.

$$\int H(t)dt \approx 1(1.3)+1(1.5)+1(2.1)+1(2.6)=6.1.$$

$$\frac{1}{4}\int H(t)\,dt \approx 1.525 \text{ MM/DAY}$$

THE PLANT GROWS AN AVERAGE OF 1.525 MM PER DAY

FROM $t=0$ TO $t=4$ DAYS.