Calculus '21-22
Integral Test

Name Solution Key

Score______________

1. Evaluate $\int_1^4 \left(\frac{x^5-4x^2-5}{x}\right) dx$ Show the antiderivative step.

$= \int_1^4 (x^4 - 4x - 5x^{-1})\,dx$

$= \frac{x^5}{5} - 2x^2 - 5\ln|x| \Big]_1^4$

$= \frac{4^5}{5} - 32 - 5\ln 4 - \left(\frac{1}{5} - 2 - 0\right)$

2. Evaluate $\int_0^{\cancel{1}\,2} x^2\sqrt{9-x^3}\,dx$. Show the u-sub and antiderivative.

$u = 9 - x^3$ $\qquad u(0) = 9$

$du = -3x^2 dx$ $\qquad u(2) = \cancel{8}\,1$

$= -\frac{1}{3}\int_9^{\cancel{8}\,1} u^{1/2}\,du$

$= \frac{-1}{3}\,\frac{u^{3/2}}{3/2}\Big]_9^{\cancel{8}\,1} = \frac{-2}{9}\left[1 - 27\right] = \frac{52}{9}$

3. Evaluate $\int_0^{\ln 2} e^x\left(2-e^x\right)^5 dx$. Show the u-sub and antiderivative.

$u = 2 - e^x \qquad u(0) = 1$

$du = -e^x dx \qquad u(\ln 2) = 0$

$= -\int_1^0 u^5 du$

$= -\frac{u^6}{6}\Big]_1^0 = \frac{1}{6}$

4. Find the average value of $y = x^2 + 2x + 4$ on $x \in [-1, 2]$. Show the u-sub and antiderivative.

$\frac{1}{2+(-1)} \int_{-1}^{2} (x^2 + 2x + 4)\, dx$

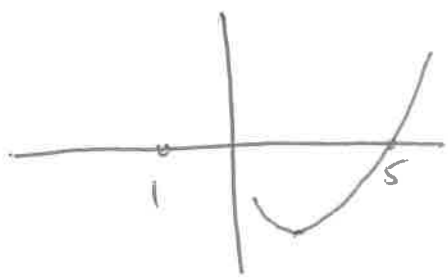
$= \frac{1}{3}\left[\frac{x^3}{3} + x^2 + 4x\right]_{-1}^{2}$

$= \frac{1}{3}\left[\left(\frac{8}{3} + 8 + 8\right) - \left(\frac{-1}{3} + 1 - 4\right)\right]$

$= 6$

5. Consider the function $y = x^2 - 4x - 5$.

a) Draw the graph of $f(x)$ on $x \in [1, 7]$.



b) Prove that the zeros of $f(x)$ are at $x = -1$ *and* 5.

$f(-1) = (-1)^2 - 4(-1) - 5 = 0$

$f(5) = 5^2 - 4(5) - 5 = 0$

or

$x^2 - 4x - 5 = (x-5)(x+1) = 0$

$x = 5, -1$

c) Find the exact value of $\int_1^7 f(x)\,dx$. Show the antiderivative and boundary insertion steps.

$$\int_1^7 (x^2-4x-5)\,dx = \left[\frac{x^3}{3}-2x^2-5x\right]_1^7$$

$$= \left(\frac{343}{3}-98-35\right)-\left(\frac{1}{3}-2-5\right)$$

$$= -12$$

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d) Find the exact area between the x-axis and $f(x)$ on $x \in [1, 7]$. Show the set-up, but solve using the calculator.

$$-\int_1^5 f(x)\,dx + \int_5^7 f(x)\,dx$$

$$= 41.333$$

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6. Let f be a differentiable function on the closed interval [1, 13] and which has values as shown on the table below.

x	1	5	10	13
$f(x)$	3	8	6	11

a) Using the sub-intervals defined by the table values, use the left-hand Riemann sum to approximate $\int_1^{13} f(x)dx$.

$$\int_1^{13} f(x)\,dx \approx 4(3) + 5(8) + 3(6)$$

$$= 70$$

b) Using the sub-intervals defined by the table values, use the trapezoidal sum to approximate $\int_1^{13} f(x)dx$

$$\int_1^{13} f(x)\,dx \approx 4\left(\frac{8+3}{2}\right) + 5\left(\frac{8+6}{2}\right) + 3\left(\frac{16+11}{2}\right)$$

$$= 82.5$$

c) Approximate $f'(10)$

$$f'(10) \approx \frac{11-8}{13-5} = \frac{3}{8}$$

7. The velocity of a particle moving along the x-axis is given by $v(t)=3t^2+10t-8$. $= (3t-2)(t+4)$

a) Find the displacement of the particle between $t=-2$ and $t=5$. Show the set-up and the antiderivative before using the calculator to find the answer.

DISPLACEMENT $= \int_{-2}^{5} v(t)\,dt$

$= t^3 + 5t^2 - 8t \Big]_{-2}^{5}$

$= 182$

b) Find the total distance traveled by the particle between $t=-2$ and $t=5$. Show the set-up but use the calculator to find the answer.

DISTANCE $= \int_a^b |v(t)|\,dt$

$= -\int_{-2}^{2/3} v(t)\,dt + \int_{2/3}^{5} v(t)\,dt$

$= 243.630$

EC. Dr. Quattrin's grandmother's family originated in the Alpine town of Sauris, Italy, where the temperature in January changes at a rate of $W(t)$ degrees Celsius per hour. $W(t)$ is a twice-differentiable, increasing and concave up function with selected values in the table below. At midnight ($t=0$), the temperature in Sauris is $-8°C$.

t (in hours after midnight)	0	1	3	6	8
$W(t)$ (in degrees Celsius per hour)	-2.6	-3.1	-1.2	1.9	2.5

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a) At approximately what rate is the rate of change of the temperature changing at 2am ($t=2$)? What would the symbol be? What would the units be?

$$W'(2) \approx \frac{-3.1-(-1.2)}{1-3} = \frac{-1.9}{2} = -.545 \ °C/hr^2$$

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b) Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 W(t)\,dt$ in the context of this problem.

$\frac{1}{8}\int_0^8 w(t)\,dt$ = THE AVERAGE TEMPERATURE CHANGE IN C° PER DAY OVER THESE 8 DAYS